Standard Course content of our Bachelor program in mathematics –

Information sheet for applicants of our Master program in Mathematics – Bonn

The following table contains short descriptions of the content of all lecture modules of the Bachelor Program in Mathematics of the University of Bonn.

This document should give you an overview about the prerequisites that are necessary for entering the Master program in mathematics. Our Bachelor students have to cover four of the six mathematical areas with at least 18, 18, 9 and 9 credit points.

Module	***Area***	***Content***
V1G1 ***Analysis I***	***Compulsory*** ***9 CP***	We discuss the axiomatic foundations of analysis and develop the theory of sequences and functions of one real variable with attention to precise definitions and rigorous proofs. In particular, we treat: convergence and limit points, series of real and complex numbers, continuity and differentiability of functions in one variable, uniform convergence. Riemann integral, partial integration and substitution formula. Power series, elementary functions (also in complex numbers), including exponential and trigonometric functions. Characteristics of elementary functions.
V1G2 ***Analysis II***	***Compulsory*** ***9 CP***	Functions of several real variables, their continuity and differentiability. Partial derivatives, especially gradient, divergence and rotation. Implicit functions theorem, norms and maps between normed spaces and connections to the convergence of functions, completeness and Banach's fixed point theorem. Ordinary differential equations, theorem of Picard-Lindelöf, solutions of linear first and second order ordinary differential equations.
V1G3 – ***Linear Algebra I*** ***(Lineare Algebra I)***	***Compulsory*** ***9 CP***	Linear Equations systems, Gaussian elimination method, groups, rings, fields (basics), vector spaces, bases and dimension, linear maps, standard scalar product on the three dimensional real space and geometric applications, matrix representation of linear maps, transformation of basis, factor spaces, determinants, Eigenvalues and Eigenvectors, characteristic polynomial, methods to diagonalize and trigonalize endomorphisms.
V1G4 ***Linear Algebra II*** ***(Lineare Algebra II)***	***Compulsory*** ***9 CP***	Jordan normal form, quadratic forms, bilinear forms, Euclidean and unitary vector spaces, principal axis transformation, symmetric maps, and geometric applications, multi-linear algebra. Following topics are optional: representation theory of chosen symmetry groups, generalized vector spaces (modules), linear optimization.
V1G5 ***Algorithmic Mathematics I*** ***(Algorithmische Mathematik I)***	***Compulsory*** ***9 CP***	**Basic algorithms and introduction to programming:** What are algorithms?, Computability, implementation of algorithms, basic concepts of programming. Introduction to programming, elementary algorithms (e.g. Euclidean algorithm), representation of numbers: integers, floating points; rounding errors; stability and complexity; sorting algorithms, **Discrete algorithms:** Graphs, trees, arborescences, connectivity, BFS and DFS, bipartite, acyclic, strongly connected graphs; linked lists; tree data structures; heaps; shortest paths, network flows, max-flow-min-cut-theorem, algorithms by Ford-Fulkerson and Edmond-Karp; bipartite matching;

Module	Area	Content
		Algorithms for solving systems of linear equations: basics: matrix norms, absolute and relative condition; algorithms: Gaussian elimination, LU-decomposition, pivoting, Cholesky-decomposition, band matrices; introduction to linearized theory of errors: forward and backward analysis, stability
V1G6 Algorithmic Mathematics II (Algorithmische Mathematik II)	***Compulsory 9 CP***	**Elementary probability theory:** concept of probability, basic models and combinatorics, expectation and variance, conditional probability, independence, weak law of large numbers, random walk, Markov chains and transition matrices. **Stochastic simulation:** pseudo random numbers, Monte Carlo method, Metropolis algorithm. **Interpolation and approximation:** interpolation: Lagrange, Hermite, divided differences, trigonometric interpolation (DFT, FFT); estimation of errors; choosing points of support; numerical integration: Newton-Cotes formulas, Romberg integration, adaptivity. **Iteration methods for large systems of linear and non-linear equations:** iterative solutions for systems of linear equations: Richardson, Jacobi, Gauss-Seidel; fixed point iteration; nonlinear minimization and calculation of zeroes: bisection method, secant method, regula falsi, Newton method (multidimensional).
V2A1 Introduction to Algebra (Einführung in die Algebra)	***A 9 CP***	Groups and rings (basics), modules of (non-commutative) rings, homological algebra, Galois theory
V2A2 Introduction to Mathematical Logic (Einführung in die Mathematische Logik)	***A 9 CP***	Syntax and semantics of predicate logic, deduction systems: term models; Gödels completeness theorem; theories and model classes; axioms of Zermelo Fraenkel set theory; formalization of basic notions in mathematics. Optional topics: deepening of sentential logic, alternative deduction systems; logical programming; Gödels incompleteness theorems; logical investigation of logical theories;
V3A1 Algebra I	***A 9 CP***	Groups and rings (deepening), field theory, Galois theory and applications.
V3A2 Algebra II	***A 9 CP***	Selected topics of algebra, for example, algebraic number theory, representation theory, commutative algebra, Lie algebras
V3A3 Foundations in Representation Theory (Grundzüge der Darstellungstheorie)	***A 9 CP***	Basic concepts of module theory, introduction to classical classification problem in representation theory
V3A4 Foundations in Number Theory (Grundzüge der Zahlentheorie)	***9 CP***	classical topics in analytic or algebraic number theory, e.g. prime number theory, zeta- and L-functions, geometry of numbers, sieve methods, arithmetic in Dedekind domains, elements of class field theory.

Module	***Area***	***Content***
V3A5 Mathematical Logic (Mathematische Logik)	***9 CP***	selected chapters of mathematical logic, e.g. model theory, set theory, computability theory.
V2B1 ***Analysis III***	***B*** ***9 CP***	**Integration theory and applications:** Lebesgue integral, especially for the n-dimensional Lebesgue-measure (also for counting measure and Dirac-measure), theorems of monotone and dominated convergence, Fubini's theorem for the Lebesgue-measure, transformation formula, surfaces in the Euclidean space and surface integrals, Gauss's and Stokes's theorems.
V2B2 ***Introduction to PDEs (Einführung in die partiellen Diffentialgleichungen)***	***B*** ***9 CP***	**Standard differential equations and classical solution methods (fundamental solutions, Fourier-Transformation):** Laplace equation: reference to electro statics (gradient and divergence), boundary value problems and their representation with the help of potentials, characteristics of harmonic functions (mean value, maximum, analyticity), Dirichlet's principle Heat equation: initial (boundary) value problem, fundamental solution, integral representation of solutions Wave equation: initial (boundary) value problem, conservation of energy, integral representation of the solution
V2B3 ***Introduction to Complex Analysis (Einführung in die komplexe Analysis)***	***B*** ***9 CP***	Holomorphic functions, power series, curve integrals, Laurent-series, Riemann's lifting theorem, essential singularities, Weierstrass's theorem of products and Mittag-Leffler's theorem, Runge- and Mergelyan approximation, Hadamard factorization theorem, Riemann's mapping theorem, prospects for the theory of several complex variables, application to selected functions, e.g. gamma-functions and elliptic functions
V3B1 ***PDEs and Functional Analysis (Partielle Differential-gleichungen und Funktionalanalysis)***	***B*** ***9 CP***	**Functional analysis:** Banach spaces and Hilbert spaces, Lax-Milgram's theorem, Sobolev spaces, in particular embedding theorems and trace theorems, weak convergence, weak sequential compactness, spectral theorem for symmetric operators with compact inverse. **Calculus of variations and elliptic PDEs:** Elliptic differential equations with nonconstant coefficients, weak and variational formulations for Dirichlet and Neumann problems, existence of minimizers, L^2-regularity theory.
V3B2 ***PDEs and Modeling (Partielle Differentiel-gleichungen und Modellierung)***	***B*** ***9 CP***	Selection of one or more of the following topics: 1. PDEs in fluid dynamics 2. PDEs for free boundary value problems and image processing 3. PDEs and mathematical physics 4. PDEs in material science
V3B3 ***Global Analysis (Globale Analysis)***	***B*** ***9 CP***	Distribution and Fourier-Transformation, oscillating integral, operators of Fourier-integrals, pseudo-differential operators, Sobolev spaces on Manifolds, embedding theorems, regularity theorem of elliptic equations on manifolds, spectral theorem for elliptic operators on closed manifolds, applications, for example, Hodge Theory.

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V2C1 Introduction to Discrete Mathematics (Einführung in die Diskrete Mathematik)	***C 9 CP***	Branchings, network flows, Goldberg-Tarjan algorithm, minimum cuts, connectivity, minimum cost flows, applications of flows and networks, bipartite matching, multi commodity flows and disjoint paths
V3C1 Linear and Integer Optimization (Lineare und Ganzzahlige Optimierung)	***C 9 CP***	Modeling of optimization problems as (integer) linear programs, polyhedron, Fourier-Motzkin elimination, Farkas' lemma, duality theorems, the simplex algorithm, network simplex, ellipsoid method, conditions for integer polyhedra, TDI systems, total unimodularity, cutting plane methods
V3C2 Combinatorics, Graphs, Matroids (Kombinatorik, Graphen, Matroide)	***C 9 CP***	Combinatorics of finite sets, basic counting theory, graphs, trees, circuits, connectivity, planarity, coloring of graphs, matroids, planar and combinatorial duality
V2D1 Introduction to Geometry and Topology (Einführung in die Geometrie und Topologie)	***D 9 CP***	Metric and topological spaces and their construction; concept of connectedness, separation axioms, compactness. Manifolds, especially surfaces and 3-manifolds, curves and surfaces in spaces, their local geometry; geodetic coverings and fundamental group.
V3D1 Topoplogy I (Topologie I)	***D 9 CP***	Singular homology groups with integer and or abitrary coefficients; axioms of homology theory, cw-complexes and cellular homology, computation of the homology for selected important spaces, such as spheres, projective spaces and surfaces. degree of maps and its application, universal coefficient theorem and Kuenneth-theorem
V3D2 Topology II (Topologie II)	***D 9 CP***	Singular cohomology groups with coefficients in commutative rings; axioms of cohomology theory. computation of the cohomology groups of some spaces, DeRahm cohomology. Universal coefficient theorem and Kuenneth-theorem. cup-product and ring structure of the cohomology. Cap-product and Poincare duality for manifolds. Higher homotopy groups, Hurewicz's theorem and Whitehead's theorem
V3D3 Foundations in Geometry and Analysis	***D 9 CP***	manifolds, tangent space, vector fields, Lie bracket and derivative, integration of vector fields, metrics, tensor calculus, connections on vector bundles, Stokes' Theorem optional (depending on preferences of the lecturer): geodesics, geodesic vs. metric completeness, de Rham cohomology,

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on Manifolds (Grundzüge der Analysis und Geometrie auf Mannigfaltigkeiten)		Theorem of Gauß-Bonnet, Poincaré Hopf Index Theorem
V3D4 Geometry (Geometrie)	***D 9 CP***	Relations between geometry and topology, symmetries
V2E1 Introduction to the Basics in Numerical Mathematics (Einführung in die Grundlagen der Numerik)	***E 9 CP***	Linear system solvers: sparse systems, gradient methods, cg, gmres, linear regression; Eigenvalue computations: vector iteration, qr-method, krylov subspace methods, svd; Numerical integration: Gaussian quadrature, more dimensional integration, Monte-Carlo methods
V2E2 Introduction to Numerical Mathematics (Einführung in die Numerische Mathematik)	***E 9 CP***	Nonlinear programming: Lagrange multipliers, KKT-systems, numerical methods; Splines: B-splines, Bezier curves, cadg; Numerical methods for ode's: one step methods, more step methods, consistency, stability
V3E1 Scientific Computing I (Wissenschaftliches Rechnen I)	***E 9 CP***	Mathematical modelling: first principles, conserved quantities, scaling aspects (dimensional analysis, homogenization, filtering); Classification of pde's and discretization: finite differences, finite elements, adaptivity, error estimators
V3E2 Scientific Computing II (Wissenschaftliches Rechnen II)	***E 9 CP***	Spectral methods, finite volume, continuous and discontinuous Galerkin, coupling of transport and diffusion, fast solvers for sparse systems, multi-grid and multi-level methods, domain decomposition. Parallelizing; strategies and load-balancing
V2F1 Introduction to Probability Theory (Einführung in die	***F 9 CP***	Probability spaces and random variables, standard models. Conditional probabilities, independence, Borel-Cantelli lemma, random walks and difference equations. Expectation, variance and co-variance. Laws of large numbers, notions of convergence for random variables. Moment generating and characteristic functions, multivariate normal distribution, central limit

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Wahrscheinlichkeits-theorie)		theorem. Basics of statistics: maximum likelihood principle, basic methods for estimation and hypothesis testing, confidence intervals. Entropy and statistical distinguishability, exponential families.
V2F2 Introduction to Statistics (Einführung in die Statistik)	***F 9 CP***	*Statistics:* parametric, non-parametric and Bayesian models, model choice, robustness. Mean square error of estimators, information inequality, connection to Fisher information and relative entropy. Level and power of hypothesis tests, Neyman-Pearson lemma. Confidence intervals and tests in Gaussian product models. Consistency of maximum likelihood, asymptotic power of likelihood quotient tests. Asymptotic normality of ML estimators. Convergence of empirical distributions, normal approximation of multinomial distribution, goodness of fit tests and their asymptotics, tests for independence, regression, analysis of variance. *Optional:* Basics of mathematical finance.
V3F1 Stochastic Processes (Stochastische Prozesse)	***F 9 CP***	Conditional expectations, conditional densities, transition probabilities. *Markov chains in discrete time:* Construction, Dirichlet problem, recurrence and transience, convergence to equilibrium, ergodicity, reversible Markov chains and Markov Chain Monte Carlo methods, Ising model. *Poisson processes and Markov chains in continuous time*, forward and backward equation. *Brownian motion:* Motivation as scaling limit of random walks, marginal laws, connection to the heat equation, existence theorem of Kolmogorov. Wiener-Lévy construction, scale invariance and symmetries, regularity of sample paths. *Large deviations:* Cramér's theorem, Sanov's theorem on finite state spaces.
V3F2 Foundations in Stochastic Analysis (Grundzüge der stochastischen Analysis)	***F 9 CP***	*Martingales:* Optional stopping, ruin problem, discrete stochastic integrals, convergence theorems, application to Markov chains, regularity and bounds for martingales in continuous time. *Itô calculus:* Quadratic variation of Brownian motion, stochastic integrals with respect to Brownian motion, Itô's formula, Martingales of Brownian motion, Lévy's characterization, stochastic representation for solutions of the Dirichlet problem and the heat equation, exit and passing times, integration with respect to Brownian semimartingales, Feyman-Kac formula, Girsanov transform.