HSM HAUSDORFF SCHOOL FOR ADVANCED STUDIES IN MATHEMATICS

UNIVERSITÄT BONN

Hausdorff School

“Special Topic School: Recent developments around joint equidistribution in number theory and dynamics”

March 10 - 14, 2025

organized by
Edgar Assing, Philippe Michel, Asbjørn Nordentoft

Abstracts

Farrell Brumley (Université Sorbonne Paris Nord)

Course 1: Duke’s equidistribution theorem

Abstract: The celebrated theorem of Duke on equidistribution of Heegner points can be seen as the starting point for many joint equidistribution problems. Duke’s proof relies on analytic tools, breaking from an earlier approach of Linnik based on ergodic theory. These methods are in fact complementary, and full consideration of each of them sets the tone for the two themes of this school. The aim of this lecture series is to discuss both approaches to Duke’s theorem in (some) detail. In so doing we shall encounter many concepts that will also play a role in the other courses. More specifically, we will formulate the statement of Duke’s theorem in a few of its classical guises, give an overview of Linnik’s ergodic approach, with an eye to understanding its famous splitting condition, and provide a suitably detailed coverage of Duke’s argument (and its extension to non-fundamental discriminants), covering quantitative aspects.

Manfred Einsiedler (ETH Zurich)

Course 2: Classificiations of joinings for higher rank actions

Abstract: The dynamics of one-parameter diagonalisable flows on homogeneous spaces is very chaotic and has no meaningful classification results of orbits, invariant measures, or their joinings. In contrast to this the dynamics of higher rank diagonalisable flows has often remarkable rigidity properties, which allow for (partial) classification results for invariant measures and joinings. We explain this difference, the joining classification by Lindenstrauss and myself, and some of the basic ideas of its proof.

Menny Aka (ETH Zurich)

Course 3: Applications of dynamics to number theory

Abstract: I will begin by discussing the following questions: How can points of spheres, Heegner points, closed geodesics, and other arithmetic objects be studied in a uniform way? Does this framework reveal relationships between these and other objects? How does their respective distribution relate to statements in homogeneous dynamics? This leads to two "pure" arithmetic problemms: the surjectivity of the reduction map from complex multiplication elliptic curves to supersingular elliptic curves (following P. Michel) and the representability of an integral quadratic form by another integral quadratic form (following Ellenberg-Venkatesh). I will describe how these problems can be reformulated as distribution problems in homogeneous spaces. Finally, I will present some natural joinings between arithmetic problems as well as some artificial ones. We will formulate conjectural distribution statements in homogeneous spaces and discuss how and when they follow from the joinings classification of Einsiedler-Lindenstrauss.

Valentin Blomer (University of Bonn)

Course 4: Joint equidistribution from the viewpoint of analytic number theory

Abstract: In this course, I will introduce techniques from analytic number theory to study various types of simultaneous equidistribution problems on the upper half plane or, more generally, quotients of quaternion algebras. The starting point is Weyl's criterion. In favorable cases, the Weyl sums can be expressed in terms of L-functions using period formulae such as Waldspurger's theorem. I will explain various techniques from multiplicative number theory, sieve theory, automorphic forms and spectral analysis to analyse such Weyl sums.

Min Lee (University of Bristol)

Course 5: Distribution of rational points on expanding horospheres: an analytic number theory approach

Abstract: The study of dynamics on homogeneous spaces has found numerous applications in number theory over the last four decades, and a new line of research is emerging: making these results effective by establishing rates of convergence for equidistribution theorems in homogeneous dynamics. In this course, we use methods from analytic number theory to investigate the equidistribution of rational points on expanding horospheres in the space of unimodular lattices in at least 3 dimensions. This problem was originally explored by Einsiedler, Mozes, Shah and Shapira, relying in particular on Ratner's measure classification results. Here, we present an alternative strategy based on automorphic forms, Fourier analysis, and exponential sums, which provides an effective estimate for the rate of convergence. The material for this course is based on joint work with D. El-Baz, B. Huang, J. Marklof, and A.Strömbergsson.

Jasmin Matz (University of Copenhagen)

Colloquium: Distribution of anisotropic tori in symplectic groups

Abstract: In 1988 Duke proved the equidistribution of Heegner points and closed geodesics on the modular surface, and one can thus ask more generally about the distribution of closed torus orbits on other locally symmetric spaces. The case of PGL(n) over arbitrary number fields was studied by Einsiedler, Lindenstrauss, Michel, and Venkatesh, and solved completely for n=3. In joint ongoing work with Farrell Brumley we now study the case of GSp(4).
