HSM HAUSDORFF SCHOOL FOR ADVANCED STUDIES IN MATHEMATICS

UNIVERSITÄT BONN

Hausdorff School

"Maximal Operators and Applications"

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organized by

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Abstracts

Jonathan Bennett (University of Birmingham)

Controlling oscillatory integrals by maximal functions

Abstract: In this series we discuss some surprising ways in which oscillatory integral operators may be controlled by maximal operators of a purely integro-geometric nature (such as maximal averaging operators). Here control takes the form of a weighted L^2 inequality of Fefferman-Stein type; that is, of the form

$$\int |Tf|^2 w \leq C \int |f|^2 Mw$$

where T is an oscillatory integral operator, and M is the "controlling" maximal operator. The primary motivation here is a conjectural inequality (often referred to as Stein's conjecture) asserting that Fourier extension operators may be controlled effectively by maximal operators of Kakeya type..

- Lecture 1: Background, beginning with historical remarks; weighted Fourier restriction theory; Stein's conjecture for the extension operator; the Mizohata-Takeuchi conjecture.
- Lecture 2: Some progress on these conjectures.
- Lecture 3: Some model problems that we understand: controlling oscillatory convolution operators by Nagel-Stein maximal operators.
- Lecture 4: Phase space formulations of the Stein and Mizohata-Takeuchi conjectures; recent approaches from X-ray tomography and optics.

Xiumin Du (Northwestern University)

Schrodinger maximal function and weighted Fourier restriction estimates

Abstract: In the first lecture, we'll introduce an almost sharp L^2 estimate for the Schrodinger maximal function, which solves Carleson's pointwise convergence problem for the free Schrodinger equation in all dimensions. This estimate about the Schrodinger maximal function fits into a more general setting, i.e. weighted Fourier restriction estimates. Such estimates have several applications in PDEs and geometric measure theory, including spherical average Fourier decay rates of fractal measures,

Falconer's distance set problem, etc. In the remaining two lectures, we'll talk about the main ingredients and strategies in the proof of an L^2 weighted Fourier restriction estimate established by Ruixiang Zhang and myself. The proof relies on an iteration of broad-narrow analysis, by invoking Bourgain-Demeter's l^2 decoupling in the narrow case and Bennett-Carbery-Tao's multilinear restriction in the broad case.

Sanghyuk Lee (Seoul National University)

Bounds on the strong spherical maximal functions

Abstract: This talk concerns L^p bounds on the strong spherical maximal functions that are multi-parametric maximal functions defined by averages over ellipsoids. We obtainL^p bounds on those maximal operators for a certain range of p. No such maximal bounds have been known until recently. In particular, our results extend Stein's celebrated spherical maximal bounds to multi-parametric versions. The talk is based on joint work with Juyoung Lee and Sewook Oh.

Malabika Pramanik (UBC)

Geometric maximal operators

Abstract: The fields of harmonic analysis, partial differential equations and geometric measure theory are home to a wide variety of maximal operators. Some are classical, dating back to Hardy and Littlewood. Others are of more recent vintage, such as those studied by Stein, Bourgain and Wolff, and their modern variants. Why are these operators everywhere? What kind of information do they encode? What type of problems do they help solve? This lecture series will offer a glimpse into the world of maximal operators of different flavours. The first talk will introduce a few operators that have seen a lot of recent activity, e.g.

- Kakeya maximal operator, and its relatives the directional maximal operators;
- Spherical/circular maximal operator and their variants;
- Maximal operators associated with measures (for example, on curves and fractals).

We will explore connections of these maximal operators with the areas like differentiation, PDE, geometric measure theory, restricted projections, and decoupling. For the remaining talks I will pick a few maximal operators that I have first-hand experience with, and devote a lecture to each. Each talk will involve one centrepiece result with applications, a proof sketch and a few tractable problems for the audience to think about.

Keith Rogers (Instituto de Ciencias Matemáticas)

Improved bounds for the Kakeya maximal operator using semialgebraic geometry

Abstract: The Kakeya problem considers thin tubes which point in different directions and how much they can be made to overlap by positioning them strategically. On the one hand, we will see that the tubes cannot be compressed too much if they are positioned in an algebraic way. The proof employs the Tarski-Seidenberg projection theorem and the Gromov-Yomdin algebraic lemma. On the other hand, polynomial partitioning can be used to show that the expected bound holds in the absence of any algebraic structure. Balancing between the two extremes yields improved bounds for the Kakeya maximal operator in higher dimensions. This is joint work with J. Hickman, N.H. Katz and R. Zhang.

Andreas Seeger (UW Madison)

The Nevo-Thangavelu spherical maximal operator

Abstract: The spherical means on the Heisenberg group were considered by Nevo and Thangavelu in order to prove maximal and pointwise ergodic theorems for certain actions of the Heisenberg groups. In these lectures I'll discuss various old and new results, and some open problems, in the context of general Carnot groups of step two. In recent works the Nevo-Thangavelu means are viewed as generalized Radon transforms associated with an incidence relation of co-dimension two or higher. Relevant background material on oscillatory integral and Fourier integral operators will be provided; no knowledge of Lie theory is required.

Christoph Thiele and Floris van Doorn (University of Bonn)

Carleson operators on doubling metric measure spaces (joint seminar)

Abstract: A Carlson operator is a maximally modulated singular integral operator. Originally motivated by convergence questions of Fourier series, more general operators have been studied including some with polynomial modulations. We introduce a set of axioms for modulation functions that allow us to define Carleson operators on doubling metric measure spaces. We will present this result, including some background (CT). We are working on formally verifying this result using the computer program Lean, and we will also discuss proof assistants and the process of verifying this result (FvD). This is joint work with Lars Becker, Floris van Doorn, Asger Jamneshan, and Rajula Srivastava.
