HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

"Differential geometry beyond Riemannian manifolds"

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organized by
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Abstracts

Bruce Kleiner (New York University)

Mean curvature flow in $\mathbb{R}^3$ and the Multiplicity One Conjecture.

Abstract: An evolving surface is a mean curvature flow if the normal component of its velocity field is given by the mean curvature. First introduced in the physics literature in the 1950s, the mean curvature flow equation has been studied intensely by mathematicians since the 1970s with the aim of understanding singularity formation and developing a rigorous mathematical treatment of flow through singularities. I will discuss progress in the last few years which has led to the solution of several longstanding conjectures, including the Multiplicity One Conjecture.
This is joint work with Richard Bamler.

Melanie Rupflin (University of Oxford)

Quantitative stability estimates for the Dirichlet energy.

Abstract: The formation of singularities poses a major difficulty in the quantitative analysis of sequences of almost critical points or almost minimisers of the Dirichlet energy of maps from surfaces to manifolds. Rather than converging to a single limiting harmonic map, energy can concentrate at multiple scales and/or near multiple points leading to the formation of a bubble tree that is given by a collection of harmonic maps which describe both the behaviour of the maps on the bulk and on the relevant micro-domains where energy concentrates. In this talk we consider how this affects the basic questions in the quantitative analysis of this variational problem, i.e. whether (and in what sense) almost minimisers/almost critical points can be expected to be close to such singular minimisers/critical points, and discuss in particular the challenges that arise from having to compare maps defined on domains of different topology and wanting to measure and control the distance between them in an optimal way.

Antoine Song (Caltech)

Geometry of the regular representation of hyperbolic groups.

Abstract: This talk is about geometric properties of the regular representation of hyperbolic groups. Given a torsion free hyperbolic group G, one can build a natural quotient Q of a Hilbert sphere from the regular representation of G. I am interested in the geometry of this infinite dimensional Riemannian space Q, and also of its ultralimit Q_ω. This ultralimit encodes the asymptotic geometry of Q, for instance possible limits of the regular representation. I will discuss some properties of both spaces. I will also mention an application: the spherical volume (a topological invariant defined by Besson-Courtois-Gallot) of a negatively curved manifold is realized by a minimal surface inside the corresponding Q_ω. This result is motivated by the problem of constructing a "minimal surface geometry" on closed manifolds.

Alexander Lytchak (KIT)

Submetries.

Abstract: Submetries are maps between metric spaces which send balls onto balls of the same radii. They generalize Riemannian submersions and quotients maps for isometric group actions. In the talk, I will discuss topological, analytic and algebraic structural results for such maps.

Richard Bamler (UC Berkeley)

Toward a theory of Ricci flow in dimension 4.

Abstract: The Ricci flow (with surgery) has proven to be a powerful tool in the study of 3-dimensional topology—its most prominent application being the verification of the Poincaré and Geometrization Conjectures by Perelman about 20 years ago. Since then further research has led to a satisfactory understanding of the flow and surgery process in dimension 3.
Recently there has been some progress on Ricci flow in higher dimensions, in the form of a new compactness and partial regularity theory. This theory relies on a new geometric perspective on Ricci flows and provides a better understanding of the singularity formation and long-time behavior of the flow. In dimension 4, in particular, this theory may eventually open up the possibility of a surgery construction or a construction of a "flow through singularities".
In the first part of the talk, will describe this new theory, the new geometric intuition that lies behind it and its implications on the study of singularities in dimension 4. In the second part, I will present new work (joint with Eric Chen) that concerns the resolution of conical singularities.

Roman Sauer (KIT)

Waist inequalities and the Kazhdan property.

Abstract: The waist inequality for the sphere by Gromov is wonderful result of geometric measure theory. Formulated appropriately for families of spaces, a (uniform) waist inequality for a family of Riemannian manifolds is the Riemannian analog of higher dimensional expanders. We show that the Kazhdan property alone gives rise not only to the Riemannian version of expanders, which is classical, but also to the one of 2-dimensional expanders. Joint with Uri Bader.

Damaris Meier (University of Fribourg)

Bubbling and homotopic energy minimizers in metric spaces.

Abstract: When searching for energy minimizers within the homotopy class of a continuous map defined on a closed Riemannian surface M into a compact Riemannian mandifold N, a phenomenon known as "bubbling" can occur in case of the target N having non-trivial second homotopy group. This was first described in the influential work of Sacks and Uhlenbeck, where they prove existence of minimal 2-spheres in such spaces N. In this talk, we will explore a novel and conceptually simple approach to the problem of showing existence of energy minimizers in given homotopy classes. This method applies to very general metric space targets X. By only assuming that X is compact, quasiconvex, and admits a local quadratic isoperimetric inequality, we obtain existence results that generalize the aforementioned work of Sacks and Uhlenbeck. This talk is based on joint work with Noa Vikman and Stefan Wenger.

Chikako Mese (Johns Hopkins University)

Harmonic Maps into Euclidean Buildings and Applications.

Abstract: In this talk, we will discuss the construction of equivariant pluriharmonic maps from the universal cover of a complex smooth quasi-projective variety X into Euclidean buildings, extending the foundational work of Gromov–Schoen to the quasi-projective setting. As an application, we examine the fundamental group of a smooth quasi-projective variety. One key consequence of the existence of a pluriharmonic map is that X admits nonzero global logarithmic symmetric differentials whenever its fundamental group has a linear representation with infinite image. Furthermore, we show that the property of smooth quasi-projective varieties having big fundamental groups is stable under small deformations. These results offer new insights into the interplay between non-Archimedean geometry, harmonic maps, and the structure of fundamental groups of quasi-projective varieties.

Christine Breiner (Brown University)

Harmonic maps into Euclidean buildings.

Abstract: We describe a regularity result for equivariant harmonic maps from the universal cover of a Riemannian manifold into a (not necessarily locally finite) Euclidean building. As an application we prove non-Archimedean superrigidity for rank 1 symmetric spaces. This result extends the work of Gromov-Schoen, who proved p-adic superrigidity by considering locally finite targets. This work is joint with B. Dees and C. Mese.

Pierre Pansu (Université Paris-Saclay)

Mass decompositions of currents.

Abstract: Can every normal current be expressed as an average of integral currents? The answer is yes in certain dimensions (0, 1, top-1 and top), but no in general. I describe a strategy for proving that every closed current is the boundary of a mass-decomposed current, which is furthermore calibrated. This is inspired by Kantorovitch and Beckmann's formulations of the optimal transportation problem. Presently, the problem is hard enough in Riemannian and subRiemannian manifolds. Later on, it will deserve being studied in more general metric spaces!

Urs Lang (ETH Zürich)

A sharp isoperimetric gap theorem in non-positive curvature.

Abstract: In joint work with Cornelia Drutu, Panos Papasoglu, and Stephan Stadler, we study isoperimetric inequalities for null-homotopies of Lipschitz 2-spheres in proper CAT(0) spaces. In one dimension less, for fillings of circles by discs, it is known that a quadratic inequality with a constant smaller than the sharp threshold $1/(4\pi)$ implies that the underlying space is Gromov hyperbolic and satisfies a linear inequality. Our main result is a first analogous gap theorem in higher dimensions, yielding exponents arbitrarily close to 1. Towards this we prove a Euclidean isoperimetric inequality for null-homotopies of 2-spheres, apparently missing in the literature, and introduce so-called minimal tetrahedra, which we demonstrate satisfy a linear inequality.

Giada Franz (Université Gustave Eiffel)

Unknottedness of free boundary minimal surfaces and self-shrinkers

Abstract: Lawson in 1970 proved that minimal surfaces in the three-dimensional sphere are unknotted. In this talk, we discuss unknottedness of free boundary minimal surfaces in the three-dimensional unit ball and of self-shrinkers in the three-dimensional Euclidean space. This is based on joint work with Sabine Chu.

Fedor Manin (University of Toronto)

The Morse landscape of the Lipschitz functional

Abstract: We can view the Lipschitz constant as a height function on the space of maps between two manifolds and ask (as Gromov did nearly 30 years ago) what its "Morse landscape" looks like: are there deep peaks, valleys, and mountain passes? A simple version of this question: given two points in the same component (homotopic maps), does a path between them (a homotopy) have to pass through maps of much higher Lipschitz constant? But we can also consider similar questions for higher-dimensional cycles in the space. In joint work with Jonathan Block and Shmuel Weinberger, we make this precise using the language of persistent homology and give some first results.

Alexander Nabutovsky (University of Toronto)

Boxing inequalities, widths and systolic geometry

Abstract: We will present generalizations the classical boxing inequality: For a bounded domain $\Omega \subset \mathbb{R}^{n+1}$ and a positive $m \in (0, n]$ $HC_m(\Omega) \leq c(m)HC_m(\partial\Omega)$, where HC_m denotes the m-dimensional Hausdorff content. Recall that $HC_m(X)$ is defined as the infimum of $\Sigma_i r_i^m$ over all coverings of X by metric balls, where r_i denote the radii of these balls. The case $m = n$ here is the classical boxing inequality that is stronger than the isoperimetric inequality.

Yet this result is only a particular case of our boxing inequality valid also in higher codimensions: For each Banach space B and compact $M \subset B$ there is a “filling" of M by W so that W is at the distance at most $c(m)HC_m^{\frac{1}{m}}(M)$ from M, and $HC_m(W) \leq const(m)HC_m(M)$. Further, the result can be generalized to the case when B is a metric space with linear contractibility function.

This result generalizes the high-codimension isoperimetric inequality for Hausdorff contents proven by B. Lishak, Y. Liokumovich, R. Rotman and the speaker originally motivated by applications to systolic geometry.

The applications to systolic geometry go via inequalities that provide upper bounds for the widths of $M \subset B$ in terms of the volume or a Hausdorff content of M. The widths $W_m^B(M)$ measure how far M

is from a m-dimensional simplicial complex in B. In the second part of the talk we will explain new inequality $W^{l^\infty}_{m-1}(M) \leq const\sqrt{m}\ vol(M^m)^{\frac{1}{m}}$ for closed manifolds $M^m \subset \mathbb{R}^N$ and its implications to systolic geometry. Here the width is measured with respect to the l^∞ distance in the ambient Euclidean space.
Joint work with Sergey Avvakumov.

Bernhard Hanke (University of Augsburg)

Lipschitz rigidity for scalar curvature

Abstract: Lower scalar curvature bounds on spin Riemannian manifolds exhibit remarkable extremality and rigidity phenomena determined by spectral properties of Dirac type operators. For example, a fundamental result of Llarull states that there is no smooth Riemannian metric on the n-sphere which dominates the round metric and whose scalar curvature is greater than or equal to the scalar curvature of the round metric, except for the round metric itself. A similar result holds for smooth comparison maps from spin Riemannian manifolds to round spheres.
In a joint work with Cecchini-Schick and Cecchini-Schick-Schönlinner, we generalize this result to Riemannian metrics with regularity less than C^1 and Lipschitz comparison maps, answering a question posed by Gromov in his "Four Lectures". To this end, we rely on a notion of scalar curvature in the distributional sense introduced by Lee-LeFloch and on spectral properties of Lipschitz Dirac operators. It turns out that the existence of a nonzero harmonic spinor field - guaranteed by the Atiyah-Singer index theorem - forces the given comparison map to be quasiregular in the sense of Reshetnyak. Thus we build an unexpected bridge from spin geometry to the theory of quasiconformal mappings.
