HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

Follow-Up-Workshop to TP

"The Next Horizon: Workshop on Open Problems in Harmonic Analysis and Analytic Number Theory"

April 28 - May 2, 2025

organized by

Valentin Blomer (Universität Bonn), Farrell Brumley (Sorbonne University), Philip Gressman (University of Pennsylvania, Philadelphia), Marina Iliopoulou (University of Athens), Lillian B. Pierce (Duke University)

Abstracts

James Maynard (University of Oxford)

Large values of Dirichlet polynomials and the zeta function

Abstract: Many important results in analytic number theory concerning primes and the zeta function depend on questions about large values of 'Dirichlet polynomials'. Often such questions are almost pure problems in harmonic analysis, and can be studied without much further number-theoretic input. Moreover, despite many different techniques for different regimes, in the many important problems there is a single limiting scenario, and here typically we are unable to improve upon a 'trivial bound'. I will highlight a few key questions of this type and the critical limiting scenario in each case.

Damaris Schindler (University of Göttingen)

Density of rational points near manifolds

Abstract: Given a bounded submanifold M in $\mathbb{R}^n$, how many rational points with common bounded denominator are there in a small thickening of M? Under what conditions can we count them asymptotically as the size of the denominator goes to infinity? I will discuss some open problems connected to these questions and explain arithmetic applications such as in Serre's dimension growth conjecture as well as applications in Diophantine approximation.

Betsy Stovall (University of Wisconsin, Madison)

Discrete vs. continuum incidence problems

Abstract: While continuum incidence problems (i.e., L^p estimates) for families of curves in $\mathbb{R}^d$ are quite well-understood, and their discrete, combinatorial analogues are natural and well-studied, there seem to be some significant obstructions both to improving current discrete methods to match the continuum case and to transferring continuum methods over to the discrete case. In this talk, we'll

look at some familiar cases where the results match, as well as the more typical scenarios where they don't.

Trevor Wooley (Purdue University)

The paucity of knowledge concerning Weyl sums and their mean values

Abstract: The past hundred or so years have witnessed tremendous progress in our understanding of exponential sums having polynomial arguments, with recent advances associated with progress on Vinogradov's mean value theorem, the delta-function method, and speculative conjectures on L-functions. Such might be the impressions of an expert in the area. Yet a critic would point out that only for linear and quadratic Weyl sums can knowledge be considered substantial. For cubic Weyl sums the situation remains highly unsatisfactory, with speculative conjectures disguising almost circular arguments. The understanding of exponential sums of high degree, meanwhile, falls far short of what is conjectured to be true. In this talk we seek to identify the common themes underlying present approaches to understanding Weyl sums, and associated obstructions to progress. There are connections with congruences to large moduli, arithmetic stratification, and speculative conjectures on L-functions.
