Report on the trimester program

Synergies between modern probability, geometric analysis and stochastic geometry

January – April 2024

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1 Topics and Goals

The main topic of the HIM trimester program was the connection between probability and geometry, with particular emphasis on geometric analysis and stochastic geometry, including e.g. random graphs, isoperimetric problems, operator norms of random matrices, random metric spaces, curvature bounds on discrete spaces, and high-dimensional statistics.

Asymptotic geometric analysis is concerned with geometric and linear properties of finite dimensional objects, when the dimension and other relevant parameters, grow to infinity. The roots of asymptotic geometric analysis are essentially in functional analysis but the area is closely tied to convex and discrete geometry and several branches of probability. An essential topic has been large deviations in asymptotic geometric analysis, which are concerned with the exponential decay of the probability of tail events, described in terms of a speed and a rate function. Large deviation theory has turned out to be a powerful device, which sheds new light on well-known geometric and analytic problems and allows to attack questions that had been out of reach with methods used so far.

Isoperimetric inequalities are in many cases reformulations of matching concentration and large deviation inequalities and thus a paradigm for the interplay between geometry and probability. Hence, isoperimetric inequalities played a key role in the trimester program. The classical isoperimetric inequality is a consequence of the Brunn-Minkowski inequality. The focus of the trimester program was on the log-Brunn Minkowski inequality and more general isoperimetric inequalities.

Stochastic geometry deals with randomly constructed sets and asks for fundamental functionals like volume, curvature or combinatorial and topological properties of the resulting set when some underlying parameters tend to infinity. Such objects appear naturally and play an essential role in mathematics and applied sciences. Both fields share common interest in many problems like random graph theory and random matrix theory, and have been strongly influenced by modern probability theory in the last decade.

In the broad field of stochastic geometry, excpectations of functionals of spatial random systems are a classical topic. More recently the description of fluctuations of especially Poisson-driven geometric random structures, i.e. variance estimates and central limit theorems have been investigated. One focus of the trimester program was on concentration for spatial random systems.

2 Activities

Essential parts of the program have been the introductory spring school and the two one-week workshops:

- Introductory Spring School, January 22.-26. 2024: The goal was to introduce large deviation theory and its applications in a geometric context. The school consisted of 3 courses with 4 lectures each given by D. Rosen (on random polytopes), C. Stegehuis (on random graphs), and K. Tatarko (an introduction to geometric analysis). The courses have been accompanied by 15 short talks by junior participants presenting their recent research results, and a poster session accompanied by blitz talks on Tuesday.

- Large deviations and (random) convex sets, February 19.-23. 2024: The main topics have been asymptotic geometric analysis and stochastic geometry and several connections between these areas. There have been 7 main lectures given by T. Tkocz, A. Litvak, B. Vritsiou, Z. Kabluchko, E. O'Reilly, R. Adamczak, and A. Gusakova. In addition we had 11 contributed talks by several participants.

- High dimensional phenomena: geometric and probabilistic aspects, March 11.-15. 2024: This workshop was dedicated to geometric and probabilistic aspects of high dimensional phenomena with a focus on probabilistic properties in high dimensional convex geometry. The main talks have been given by L. Sauermann, A. Colesanti, M. Ludwig, G. Livshyts, M. Rudelson, D. Hug, V. Yaskin, A. Naor, S. Vempala, M. Fradelizi, A. Koldobsky, and A. Bernig, and with additional 13 contributed talks by participants.

In addition, a seminar series was organized each Tuesday where participants presented their recent research projects with a focus on questions and discussions. In total around 60 participants attended the trimester program, collaborated on different thematic subjects, and benefited from the open discussions in an inviting and pleasant atmosphere.

3 Results

The following list contains preprints and published articles by March 2025 which mention the trimester program in the acknowledgements.

[1] A. Arman, A.E. Litvak. Minimal dispersion on the cube and the torus. Journal of Complexity 85 (2024) 101883
https://doi.org/10.1016/j.jco.2024.101883

[2] D. Beck. The Probability that a Random Triangle in a Cube is Obtuse.
https://doi.org/10.48550/arXiv.2501.11611

[3] D. Beck. On Random Simplex Picking Beyond the Blaschke Problem.
https://doi.org/10.48550/arXiv.2412.07952

[4] D. Beck, Z. Lv, A. Potechin. On the second moment of the determinant of random symmetric, Wigner, and Hermitian matrices.
https://doi.org/10.48550/arXiv.2409.14620

[5] F. Besau, A. Gusakova, C. Thäle. Random polytopes in convex bodies: Bridging the gap between extremal containers.
https://doi.org/10.48550/arXiv.2411.19163

[6] F. Besau, E.M. Werner. The L_p-floating area, curvature entropy, and isoperimetric inequalities on the sphere.
https://doi.org/10.48550/arXiv.2411.01631

[7] L. Brauner, Georg C. Hofstätter, O. Ortega-Moreno. The Klain approach to zonal valuations.
https://doi.org/10.48550/arXiv.2410.18651

[8] J. de Dios Pont. Convex sets can have interior hot spots.
https://doi.org/10.48550/arXiv.2412.06344

[9] J. Haddad, D. Langharst, G. Livshyts, E. Putterman. On the polar of Schneider's difference body.
https://doi.org/10.48550/arXiv.2503.06191

[10] D. Hug, G. Last, M. Schulte. Boolean models in hyperbolic space.
https://doi.org/10.48550/arXiv.2408.03890

[11] D. Hug, F. Mussnig, J. Ulivelli. Additive kinematic formulas for convex functions. Canadian Journal of Mathematics. 2025;1–23.
https://doi.org/10.4153/S0008414X24000944

[12] D. Langharst, F. Marin Sola, J. Ulivelli. Higher-Order Reverse Isoperimetric Inequalities for Log-concave Functions.
https://doi.org/10.48550/arXiv.2403.05712

[13] D. Langharst, E. Putterman. Weighted Berwald's Inequality.
https://doi.org/10.48550/arXiv.2210.04438

[14] R. Latala. On the spectral norm of Rademacher matrices.
https://doi.org/10.48550/arXiv.2405.13656

[15] S. Myroshnychenko, C. Tang, K. Tatarko, T. Tkocz. Stability of simplex slicing.
https://doi.org/10.48550/arXiv.2403.11994

[16] J. Prochno, C. Schütt, M. Sonnleitner, E. M. Werner. Random approximation of convex bodies in Hausdorff metric.
https://doi.org/10.48550/arXiv.2404.02870

[17] Y.W. Qiu. Some super-Poincaré inequalities for gaussian-like measures on stratified Lie groups.
https://doi.org/10.48550/arXiv.2404.00293

[18] M. Reitzner, A. Strotmann. Poisson-Delaunay approximation.
https://doi.org/10.48550/arXiv.2410.23003

[19] W. Rui Sun, B.-H. Vritsiou. On the illumination of 1-symmetric convex bodies.
https://doi.org/10.48550/arXiv.2407.10314

[20] W. Rui Sun, B.-H. Vritsiou. Illuminating 1-unconditional convex bodies in $\mathbb{R}^3$ and $\mathbb{R}^4$, and certain cases in higher dimensions.
https://doi.org/10.48550/arXiv.2407.11331

[21] H. Sambale, C. Thäle, T. Trauthwein. Central limit theorems for the nearest neighbour embracing graph in Euclidean and hyperbolic space.
https://doi.org/10.48550/arXiv.2411.00748