HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS

UNIVERSITÄT BONN

"Workshop: High dimensional phenomena: geometric and probabilistic aspects"

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organized by
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Abstracts

Rotem Assouline (Weizmann Institute of Science)

Brunn-Minkowski inequalities for sprays on surfaces

Abstract: We propose a generalization of the Minkowski average of two subsets of a Riemannian manifold, in which geodesics are replaced by an arbitrary family of parametrized curves. Under certain assumptions, we characterize families of curves on a Riemannian surface for which a Brunn-Minkowski inequality holds with respect to a given volume form. In particular, we prove that under these assumptions, a family of constant-speed curves on a Riemannian surface satisfies Brunn-Minkowski with respect to the Riemannian area form if and only if the geodesic curvature of its members is determined by a function κ on the surface, and κ satisfies the inequality $\mathrm{K}+\kappa^2 - |\nabla\kappa| \geq 0$, where K is the Gauss curvature.

Andreas Bernig (Goethe Universität Frankfurt a. M.)

Hard Lefschetz theorem and Hodge-Riemann relations for convex valuations

Abstract: It is shown that the algebra of smooth valuations satisfies a set of properties well known in Kähler geometry, algebraic geometry and combinatorics, namely a mixed hard Lefschetz theorem and mixed Hodge-Riemann relations. They can be translated into quadratic inequalities for mixed volumes of sufficiently smooth convex bodies that generalize the classical Alexandrov-Fenchel inequality. The same quadratic inequalities were known to hold for the summands of a simple polytope by work of McMullen, and they were known to fail for arbitrary (non-smooth) convex bodies by a counter-example due to van Handel. Our proof uses perturbation theory of unbounded linear operators. Joint work with Jan Kotrbatý and Thomas Wannerer.

Florian Besau (Vienna University of Technology)

Floating bodies and duality in spaces of constant curvature

Abstract: Meyer & Werner showed that Lutwak's p-affine surface area in d-dimensional Euclidean space arises as the volume derivative of the floating body of convex body conjugated by polarity for

$p = -d/(d+2)$. We establish an extension of this relation in the spherical and hyperbolic space. Our results hold in spaces of constant curvature, and we also show that the Euclidean result of Meyer & Werner can be obtained by a limiting process as the space curvature tends to zero. Based on joint work with Elisabeth Werner.

Leo Brauner (Vienna University of Technology)

Lefschetz operators on Minkowski valuations

Abstract: Minkowski valuations are finitely additive operators on the space of convex bodies. They form a rich class of geometric maps, including the difference body, projection body, and mean section body maps. The Lefschetz operators allow us to move between valuations of different degrees of homogeneity. In this talk, we discuss the action of the Lefschetz operators on continuous Minkowski valuations that are compatible with rigid motions. This is joint work with Georg C. Hofstätter and Oscar Ortega-Moreno.

Andrea Colesanti (University of Florence)

Variational functionals in the Gauss space

Abstract: Three classical functionals of mathematical physics, the electrostatic capacity, the torsion and the first Dirichlet eigenvalue of the Laplace operator, present several features which are particularly interesting by the point of view of Convex Geometry, like Brunn-Minkowski type inequalities, Hadamard formulas for their first variation, and well-posedness of the Minkowski problem. These functionals admit a natural extension to the Gauss space. I will present some results and problems, mainly concerning the possibility to prove inequalities of Brunn-Minkowski type for these extended functionals. Based on collaborations with E. Francini, G. Livshyts and P. Salani, and with L. Qin.

Maria Alfonseca Cubero (North Dakota State University)

On uniqueness questions of Croft, Falconer and Guy

Abstract: We consider a convex body K and a collection of hyperplanes which satisfy one (or more) of the following conditions: Condition (V): All hyperplanes cut off the same constant volume from K Condition (A): All hyperplane sections of K have equal area Condition (I): All hyperplane sections of K have equal moments of inertia Condition (H): All the hyperplanes are at the same distance from the origin etc... Croft, Falconer and Guy asked whether any two conditions imply that K must be the Euclidean ball. However, for several pairs of these conditions, the answer has been shown to be negative. In this talk we consider three conditions (V,I,H) and (V,A,H), or else two conditions from the list with additional normalization hypotheses on K, and give positive results for these cases. This is joint work with D. Ryabogin, A. Stancu and V. Yaskin.

Dima Faifman (Tel Aviv University)

Around the Nash problem for smooth valuations

Abstract: Smooth valuations on manifolds were introduced by Alesker as a generalization of smooth valuations on convex sets. We show that, essentially, all smooth valuations on a manifold can be obtained by restricting the translation-invariant smooth valuations in a sufficiently high-dimensional linear space, following a carefully chosen embedding of the manifold therein. We deduce the existence of Crofton formulas for all smooth valuations on manifolds. We will then consider a counterpoint problem with finite families of subspaces replacing the manifold, uncovering a surprising flexibility property of translation invariant valuations. Based on a joint work with Georg Hofstaetter.

Matthieu Fradelizi (Université Gustave Eiffel)

Entropy, isotropic constant and Mahler's conjecture

Abstract: I shall present functional forms of the work of Bo'az Klartag who used projective transformations to show that, given a convex body K with minimal volume product, its isotropic constant L_K is related to its volume product $P(K)$. As a corollary, he obtained that a strong form of the slicing conjecture (the conjecture that L_K is maximal for simplices) implies Mahler's conjecture. For log-concave functions, we adapt his ideas and need to introduce more movements to get the analogue results. Moreover, the results hold only if we choose the suitable version of the isotropic constant, the one involving the entropy. We also consider and compare the various strong forms of the slicing conjecture for s-concave functions. Based on a work in collaboration with Francisco Marin Sola.

Julian Haddad (University of Sevilla)

Fiber symmetrization in the space of matrices

Abstract: Fiber symmetrization is a process defined by Bianchi, Gardner and Gronch, that generalizes Steiner symmetrization to higher dimensions, replacing one dimensional lines by m-dimensional subspaces. Its application in the space of $n \times m$ real matrices with respect to some particular m-dimensional subspaces, has interesting properties related to matrix multiplication. I'll review some of these properties, and use it to prove some isoperimetric inequalities for convex bodies in the matrix space.

Orli Herscovici (St. John's University)

Gaussian B-inequality: stability and equality cases

Abstract: In this talk we consider the stability and equality cases in the Gaussian B-inequality. The talk is based on a joint work with Galyna Livshyts, Liran Rotem, and Alexander Volberg.

Daniel Hug (Karlsruhe Institute of Technology)

Boolean models in hyperbolic space

Abstract: The union of the particles of a stationary Poisson process of compact (convex) sets in Euclidean space is called Boolean model and is a classical topic of stochastic geometry. In this paper, Boolean models in hyperbolic space are considered, where one takes the union of the particles of a stationary Poisson process in the space of compact (convex) subsets of the hyperbolic space. Geometric functionals such as the volume of the intersection of the Boolean model with a compact convex observation window are studied. In particular, the asymptotic behavior for balls with increasing radii as observation windows is investigated. Exact and asymptotic formulas for expectation, variances and covariances are shown and univariate and multivariate central limit theorems are derived. Compared to the Euclidean framework, some new phenomena can be observed.

Jonas Knoerr (Vienna University of Technology)

Complex Monge-Amp‘ere operators and functional Hermitian intrinsic volumes

Abstract: I will present a geometric construction of certain Monge-Amp‘ere-type operators defined on the space of finite convex functions. For convex functions on Cn, this naturally gives rise to three distinct families of equivariant operators with rather different integrability and continuity properties. In addition, they differ substantially in their behavior under restrictions to subspaces. We will discuss how these properties are reflected in the valuations constructed from these operators, which we propose as functional versions of the Hermitian intrinsic volumes, which where originally introduced by Bernig and Fu. In fact, these functionals provide a complete description of the space of U(n)-invariant, continuous, and dually epi-translation invariant valuations on convex functions. In this sense, these valuations are the Hermitian analog of the functional intrinsic volumes introduced by Colesanti, Ludwig, and Mussnig.

Alexander Koldobsky (University of Missouri-Columbia)

Comparison problems for the Radon transform.

Abstract: Given two non-negative functions f and g such that the Radon transform of f is pointwise smaller than the Radon transform of g, does it follow that the L^p-norm of f is smaller than the L^p-norm of g for a given $p > 1$? We consider this problem for the classical and spherical Radon transforms. In both cases we point out classes of functions for which the answer is affirmative, and show that in general the answer is negative if the functions do not belong to these classes. The results are in the spirit of the solution of the Busemann-Petty problem from convex geometry, and the classes of functions that we introduce generalize the class of intersection bodies introduced by Lutwak in 1988. We also deduce slicing inequalities that are related to the well-known Oberlin-Stein type estimates for the Radon transform. This is joint work with Michael Roysdon and Artem Zvavitch.

Dylan Langharst (Sorbonne University)

Higher-order affine isoperimetric inequalities

Abstract: Schneider generalized the difference body of a convex body to higher-order, and also established the higher-order analogue of the Rogers-Shephard inequality. In this talk, we extend this idea to the projection body, centroid body, LYZ Body, and radial mean bodies, as well as prove the

associated inequalities (analogues of Zhang's projection inequality, Petty's projection inequality, the Busemann-Petty centroid inequality, Busemann's random simplex inequality and the affine Sobolev inequality). Joint work with J. Haddad, E. Putterman, M. Roysdon and D. Ye.

Galyna Livshyts (Georgia Institute of Technology)

Geometric functional inequalities, old and new

Abstract: We discuss a general scheme that allows to realize certain geometric functional inequalities as statements about convexity of some functionals, and inspired by the work of Bobkov and Ledoux, we obtain various interesting inequalities as their realizations. For example, we draw a link between Ehrhard's inequality and Bobkov's inequality, and several new and more general inequalities are discussed as well. In this talk we discuss a joint project with Barthe, Cordero-Erausquin and Ivanisvili, and also mention briefly some results from a joint project with Cordero-Erausquin and Rotem.

Monika Ludwig (Vienna University of Technology)

Recent advances in valuations on function spaces

Abstract: A functional Z defined on a space of real-valued functions F is called a valuation if $Z(f \vee g) + Z(f \wedge g) \equiv Z(f) + Z(g)$ for all $f, g \in F$ such that the pointwise maximum $f \vee g$ and the pointwise minimum $f \wedge g$ are in F. The important classical notion of valuations on convex bodies in $\mathbb{R}^n$ is a special case of the rather recent notion of valuations on function spaces. We present new results on valuations on spaces of convex functions on $\mathbb{R}^n$ and continuous functions on $\mathbb{S}^{n-1}$. In particular, we discuss classification results for measure-valued valuations.

Assaf Naor (Princeton)

Quantitative Wasserstein rounding.

Abstract: The main focus of this talk will be to describe recent work (joint with Braverman) on the Lipschitz extension problem that obtains solutions to various natural quantitative questions by thinking about its (known) dual formulation as a question about randomly rounding an ambient metric space to its subset while preserving certain natural guarantees that are measured in terms of transportation cost. We will start by discussing the classical formulation of these old questions as well as some background and earlier results, before passing to examples of how one could reason quantitatively using the dual perspective.

Eli Putterman (Tel Aviv University)

Affine isoperimetry, affine Sobolev inequalities, and random polytopes

Abstract: Petty's projection inequality is an affine-invariant strengthening of the isoperimetric inequality, and was used by Zhang to obtain an affine-invariant, strengthened version of the classical Sobolev inequality,. The recently-developed theory of higher-order projection bodies allows one to obtain a different affine-invariant strengthening of the isoperimetric inequality, which leads to a different affine Sobolev inequality. However, it is not a priori clear whether the higher-order affine isoperimetric/Sobolev inequalities are stronger than, weaker than, or incomparable to the classical Petty projection and affine Sobolev inequalities. In this talk we give a partial answer to this question by exhibiting a family of convex bodies for which the new higher-order affine isoperimetric inequalities are asymptotically stronger than the classical ones. As we shall see, the main computation in the proof reduces to a new question in the theory of random polytopes: giving a small-ball estimate for the mean width, i.e., bounding the probability that the mean width of a certain random polytope is a small fraction of its expectation.

Mark Rudelson (University of Michigan)

Minimal rank of submatrices of a random rectangular matrix.

Abstract: Consider an n by N matrix with independent identically distributed random entries. If N is close to n, then all square submatrices of this matrix are invertible with high probability. However, if N is much larger than n, this is no longer the case. We prove a conjecture of Feige and Lellouche regarding the minimal rank of all such square submatrices.

Lisa Sauermann (Bonn University)

On the extension complexity of random polytopes

Abstract: It is sometimes possible to represent a complicated polytope as a projection of a much simpler polytope. To quantify this phenomenon, the extension complexity of a polytope P is defined to be the minimum number of facets in a (possibly higher-dimensional) polytope from which P can be obtained as a (linear) projection. This notion plays a role in combinatorial optimization. In this talk, we discuss results on the extension complexity of random d-dimensional polytopes for fixed dimension d (obtained as convex hulls of random points on either on the unit sphere or in the unit ball). Joint work with Matthew Kwan and Yufei Zhao.

Maud Szusterman (Tel Aviv University)

Tuning Banaszczyk's transform with respect to the Gaussian measure

Abstract: Banaszczyk proved (1998) that given a convex body K with gaussian measure p 1/2, and given an arbitrary sequence of vectors from the unit ball, one can balance the sum of these vectors so that it lies in 5K. The proof relies on the monotonicity of the gaussian measure of convex bodies under a certain transform (known as Banaszczyk's transform). In this talk, we provide a new, more simple proof of this monotonicity, relying on a reduction to half-planes. We show that monotonicity holds for arbitrary convex bodies, at the cost of a doubly exponential rescaling near p=0. Our results imply a negative answer to a strong form of a vector balancing conjecture due to Banaszczyk and Szarek. Joint work with P. Nayar.

Tara Trauthwein (University of Oxford)

CLTs for Poisson functionals: the Malliavin-Stein Approach

Abstract: Common questions in stochastic geometry include the following: given a quantity derived from a random collection of points, can we show a central limit theorem? In this talk, we will show how the Malliavin-Stein method allows us to do just this by controlling what happens when we add a point (or two). We will also see how to derive a CLT for the specific example of the Online Nearest Neighbour Graph, using some recent reduction of the moment conditions necessary to apply the method.

Hirsoshi Tsuji (University of Osaka)

The functional volume product under heat flow

Abstract: This talk is based on a joint work with Shohei Nakamura (Osaka University / University of Birmingham). In this talk, we report the monotonicity of the functional volume product along the Fokker–Planck heat flow. To see this monotonicity, we consider the regularized functional volume product which is motivated from the work by Bobkov–Gentil–Ledoux. As a result, we obtain not only the monotonicity of the functional volume product, but also an improved Borell's hypercontractivity and the lower bound for a quasi-norm of the Laplace transform.

Santosh Vempala (Georgia Tech)

Beyond Moments: Robust certificates for affine transformations

Abstract: Suppose you are given random samples from an unknown affine transformation of a standard unit hypercube $[0,1]^n$ with up to an ϵ fraction of the samples arbitrarily corrupted (i.e., an adversary can replace them with any points of their choice). Can you still recover the transformation approximately? In this talk, we will present a polytime algorithm for this problem, which recovers the transformation and the uncorrupted distribution to within $O\epsilon$ total variation distance, asymptotically the best possible guarantee. As part of the analysis, we give a robust certificate for the affine transformation. Unlike previous work in robust statistics, it turns out that this problem cannot be solved using the method of moments. i.e., having any constant moment to within ϵ error does not suffice to recover the underlying transformation. We will highlight a few perplexing open problems. This is joint work with He Jia and Pravesh Kothari.

Vlad Yaskin (University of Alberta)

Some problems about centroids of convex bodies

Abstract: In this talk we will discuss some recent progress on problems concerning centers of mass of convex bodies.
