JUNIOR TRIMESTER PROGRAM
ALGEBRAIC GEOMETRY: DERIVED CATEGORIES, HODGE THEORY, AND CHOW GROUPS
HAUSDORFF RESEARCH INSTITUTE FOR MATHEMATICS
FINAL REPORT

SARAH FREI

Group: HyperK3s and special cubic fourfolds

Group members: Anna Abasheva, Corey Brooke, Sarah Frei, Richard Haburcak, Tomohiro Karube, Kohei Kikuta, Pablo Magni, Lisa Marquand, Luigi Martinelli, Genki Ouchi, Jack Petok, Xuqiang Qin

Group leaders: Sarah Frei, Richard Haburcak, Genki Ouchi

Duration: September 11 - December 20, 2023

Scientific activities

There were two weekly seminar series associated with the JTP, organized by Raymond Cheng, Sarah Frei, Mirko Mauri, and Laura Pertusi. Members of this group attended and gave talks in those seminars. Additionally, members of this group organized a summer school at the start of the program, a weekly learning seminar, and a mini-course. These activities are discussed further below.

School. The Summer School *K3 surfaces, hyperkähler manifolds, and cubic fourfolds* was organized by Sarah Frei, Richard Haburcak, Genki Ouchi, Jack Petok, and Xuqiang Qin, and took place September 11 - 15, 2023. The goal of the school was to introduce participants to the tools and techniques used in the study of K3 surfaces, hyperkähler manifolds, and cubic fourfolds. The talks were accessible to graduate students in the research area. There were 54 participants with four expository mini-courses given by senior participants and four short introductory talks from junior participants.

The mini-courses were:

- *Finite group actions on hyperkähler manifolds*, Chiara Camere (University of Milan)
- *Brauer groups and twisted sheaves on K3 surfaces*, Daniel Huybrechts (University of Bonn)
- *Lagrangian Fibrations of Holomorphic Symplectic Varieties*, Christian Lehn (University of Technology Chemnitz)
- *Lieblich-Olsson deformation technique and applications*, Sofia Tirabassi (Stockholm University)

The introductory talks were:

Date: December 31, 2024.

- *A geometrical description for birational involutions on an infinite number of Hilbert schemes of points on K3 surfaces*, Pietro Beri (Université Paris Cité)
- *Hyperkähler manifolds from cubic fourfolds*, Franco Giovenzana (Université Paris-Saclay)
- *On symplectic transformations of OG10-type hyperkähler manifolds via cubic fourfolds*, Stevell Muller (Saarland University)
- *The defect of a cubic threefold*, Sasha Viktorova (KU Leuven)

All notes and videos from the Summer School are available at the JTP website.

Learning seminar. The weekly learning seminar *Selected Topics Related To Cubics and Hyperkählers* (STReTCH) was organized by Jack Petok and Xuqiang Qin. It focused on understanding various hyperkähler manifold constructions, such as the construction of Laza-Saccà-Voisin and of the LLSvS eightfold, and then covered various topics of importance in the study of cubic hypersurfaces. Participants gave introductory and expository talks on these topics. The schedule of talks can be found at the seminar website.

Mini-course. The group members invited Emma Brakkee from Leiden University to give a mini-course titled *K3 surfaces and modular curves*. The mini-course consisted of three lectures, given on November 27, 28, and 30, 2023. The goal of the course was to explain how modular curves can be used to approach questions about K3 surfaces of both arithmetic and of derived nature.

Outcomes

The general research interests of our group were hyperkähler manifolds, K3 surfaces, and cubic fourfolds. More specifically, a sampling of the research conducted during the program is the following: birational geometry of Fano varieties of lines on cubic fourfolds, Brill-Noether theory and K3 surfaces, K3 categories, constructions of lax stability conditions on K3 surfaces, automorphism group of cubic fourfolds and K3 surfaces, quantum differential equations and semiorthogonal decompositions, and derived equivalences of hyperkähler varieties.

Here is a list of preprints that were worked on or initiated during the program at the Hausdorff Institute by the members of the group (the group members are in bold).

[AHL24] Auel, A., **Haburcak, R.**, and Larson, H. Maximal Brill–Noether loci via the gonality stratification.

[AHK24] Auel, A., **Haburcak, R.** and Knutsen, A.L. Distinguishing Brill–Noether loci.

[AP24] Auel, A., and **Petok, J.**. Zeta functions of K3 categories over finite fields.

[ACC+24] Arena, V., Canning, S., Clader, E., **Haburcak, R.**, Li, A.Q., Mok, S.C., and Tamborini, C. Holomorphic forms and non-tautological cycles on moduli spaces of curves.

[BBF+24] Belmans, P, Bose, J., **Frei, S.**, Gould, B., Hotchkiss, J., Lamarche, A., **Petok, J.**, Rodriguez Avila, C. and Shah, S. On decompositions for Fano schemes of intersections of two quadrics.

[BFMQ24] **Brooke, C., Frei, S., Marquand, L.**, and **Qin, X.** Birational geometry of Fano varieties of lines on cubic fourfolds containing pairs of cubic scrolls.

[BH24] Bud, A., and **Haburcak, R.** Maximal Brill–Noether loci via degenerations and double covers.

[Kar24] **Karube, T.** The noncommutative MMP for blowup surfaces.

[Kik24] **Kikuta, K.** Geometrical finiteness for automorphism groups via cone conjecture.

References

[ACC+24] Veronica Arena, Samir Canning, Emily Clader, Richard Haburcak, Amy Q. Li, Siao Chi Mok, and Carolina Tamborini. Holomorphic forms and non-tautological cycles on moduli spaces of curves, 2024. Available at https://arxiv.org/abs/2402.03874.

[AHK24] Asher Auel, Richard Haburcak, and Andreas Leopold Knutsen. Distinguishing Brill-Noether loci, 2024. Available at https://arxiv.org/abs/2406.19993.

[AHL24] Asher Auel, Richard Haburcak, and Hannah Larson. Maximal Brill–Noether loci via the gonality stratification, 2024. Available at https://arxiv.org/abs/2310.09954.

[AP24] Asher Auel and Jack Petok. Zeta functions of K3 categories over finite fields, 2024. Available at https://math.dartmouth.edu/~auel/papers/docs/AP_zeta_K3_category.pdf.

[BBF+24] Pieter Belmans, Jishnu Bose, Sarah Frei, Benjamin Gould, James Hotchkiss, Alicia Lamarche, Jack Petok, Cristian Rodriguez Avila, and Saket Shah. On decompositions for Fano schemes of intersections of two quadrics, 2024. Available at https://arxiv.org/abs/2403.12517.

[BFMQ24] Corey Brooke, Sarah Frei, Lisa Marquand, and Xuqiang Qin. Birational geometry of fano varieties of lines on cubic fourfolds containing pairs of cubic scrolls, 2024. Available at https://arxiv.org/abs/2407.18904.

[BH24] Andrei Bud and Richard Haburcak. Maximal Brill-Noether loci via degenerations and double covers, 2024. Available at https://arxiv.org/abs/2404.15066.

[Kar24] Tomohiro Karube. The noncommutative MMP for blowup surfaces, 2024. Available at https://arxiv.org/abs/2410.18446.

[Kik24] Kohei Kikuta. Geometrical finiteness for automorphism groups via cone conjecture, 2024. Available at https://arxiv.org/abs/2406.18438.