Report of
Laura Pertusi
group leader of

Derived categories (in algebraic geometry)

in the Hausdorff Junior Trimester Program
Algebraic geometry: derived categories, Hodge theory, and Chow groups.

Group members Soheyla Feyzbakhsh (Imperial College London); Qingyuan Jiang (University of Edinburgh); Martin Kalck (University of Graz); Johannes Krah (Universität Bielefeld); Dion Leijnse (Universiteit van Amsterdam); Shengxuan Liu (University of Warwick); Zhiyu Liu (Zhejiang University); Luigi Martinelli (Université Paris-Saclay); Nebojsa Pavic (Leibniz Universität Hannover); Laura Pertusi (Università degli Studi di Milano); Franco Rota (University of Glasgow); Fei Xie (University of Edinburgh); Shizhuo Zhang (Max-Planck institute for mathematics); Xiaolei Zhao (University of California Santa Barbara).

Organized activities During the program Martin Kalck, Shizhuo Zhang and I have co-organized the "Derived workshop" from November 6 to 10, 2023. The study of derived categories has a central role in algebraic geometry. A celebrated theorem by Bondal and Orlov states that smooth projective varieties with ample (anti)canonical bundle and equivalent bounded derived categories are isomorphic. This statement is not true in general, but weaker formulations involving the notion of semiorthogonal decomposition have been proved in many interesting geometric contexts. Furthermore, moduli spaces of stable objects in the derived category provide a way to construct new smooth projective varieties and to study their birational geometry by wall-crossing. In the framework of resolutions of singularities, derived categories can be used to characterize the singularity of a variety. The aim of this workshop was to bring together experts working in these fields to discuss recent developments in the theory and exchange new ideas.

Joint with Raymond Cheng, Sarah Frei and Mirko Mauri, I have co-organized the trimester seminar series, consisting of research seminars, called MAGHI and SAG, from participants to the semester or invited experts.

Together with the members of my group, I have co-organized the following mini-courses on selected research topics concerning the interests of the group members, held by group members and invited senior researchers:

- "Shifted Symplectic Structures" by Hyeonjun Park (Korea Institute for Advanced Study);
- "Derived categories and singularities" by Nebojsa Pavic (Leibniz Universität Hannover);
- "Quasi-BPS categories" by Tudor Padurariu (MPIM Bonn) and Yukinobu Toda (Kavli IPMU, University of Tokyo);

- "Sheaves on hyperkähler manifolds and deformation theory" by Alessio Bottini (Università di Roma Tor Vergata).

Research impact The Junior Trimester Program was a great opportunity to create new collaborations and improve our knowledge in different research directions. The following papers originated by collaborations of the group members during the JTP:

- Lin, X.; Zhang, S., Serre algebra, matrix factorization and categorical Torelli theorem for hypersurfaces, arXiv:2310.09927.
- Bayer, A.; Chen, H.; Jiang, Q., Brill–Noether Theory of Hilbert Schemes of Points on Surfaces, International Mathematics Research Notices (2023).
- Hu, X.; Krah, J., Autoequivalences of Blow-Ups of Minimal Surfaces, Bull. London Math. Soc. (2024).
- Li, C.; Liu, S. A Note on Spherical Bundles on K3 Surfaces, arXiv:2310.10842.
- Dell, H.; Jacovskis, A.; Rota, F., Cyclic covers: Hodge theory and categorical Torelli theorems, arXiv:2310.13651.
- Fan, C.; Liu, Z.; Ma, S.K., Stability manifolds of Kuznetsov components of prime Fano threefolds, arXiv:2310.16950.
- Moschetti, R.; Rota, F.; Schaffler, L., The non-degeneracy invariant of Brandhorst and Shimada families of Enriques surfaces, arXiv:2309.14981.
- Chen, H; Pertusi, L.; Zhao, X, Some remarks about deformation theory and formality conjecture, Ann. Univ. Ferrara (2024).

We would like to thank the administration of the Junior Trimester Program for their valuable support and for providing us a stimulating work environment and plenty opportunities to initiate scientific discussions and collaborations.