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CENTER FOR MATHEMATICS

UNIVERSITÄT BONN

Workshop

"Representations of supergroups"

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organized by
Thorsten Heidersdorf, Catharina Stroppel

Abstracts

Sigiswald Barbier (Ghent University)

Minimal representations and unitary representations for Lie supergroups *(short talk)*

Abstract: Lie supergroups are a generalization of Lie groups inspired by supersymmetry. As such, the study of the unitary representations of a Lie supergroup is an important problem. However, the first major obstacle is the lack of a clear and appropriate definition of what a super unitary representation should be. Various definitions have been proposed, but none offer an adequate or entirely satisfactory solution. Using minimal representations as a guideline, I will give in this talk an overview of the different approaches to unitary representations for Lie supergroups. These minimal representations are a special class of 'small' infinite-dimensional representations I have constructed with various coauthors. For (non-super) Lie groups, minimal representations are the ones that correspond to the minimal nilpotent coadjoint orbit under the orbit method.

Lukas Bonfert (Max-Planck-Institut für Mathematik)

The Weyl groupoids of $sl(m|n)$ and $osp(r|2n)$ *(short talk)*

Abstract: For Lie superalgebras the Weyl group is not as powerful as for Lie algebras, for instance it fails to permute the Borel subalgebras. To fix this it has been proposed that the Weyl group should be replaced by a "Weyl groupoid", which also includes the odd reflections. In my talk I will explain how these ideas are related to the notion of Weyl groupoid introduced by Heckenberger and Yamane (2008) in the theory of Nichols algebras, and I will provide an explicit combinatorial description of the Weyl groupoid for $sl(m|n)$ and $osp(r|2n)$. If time permits, I will also sketch the relation to Serganova's definition of Weyl groupoid and the root groupoid recently introduced by Gorelik, Hinich and Serganova. This talk is based on joint work with Jonas Nehme (arXiv:2305.04751)

John Brundan (University of Oregon)

Isomeric Heisenberg and Kac-Moody categorification

Abstract: The isomeric Heisenberg category acts naturally on a number of abelian categories appearing in the representation theory of the isomeric supergroup Q(n), and also on representations of Sergeev's algebra which is related to the double covers of symmetric groups. I will explain an efficient way to convert an action of the isomeric Heisenberg category on these and other abelian categories into an action of a corresponding super Kac-Moody 2-category. To properly understand the odd simple root indexed by the element zero of the ground field requires the theory of odd symmetric functions developed by Ellis, Khovanov and Lauda.

Shun-Jen Cheng (Academia Sinica)

Super Duality of Whittaker Modules and Finite W-(super)algebras

Abstract: We explain an equivalence of categories of Whittaker modules over Lie algebras and Lie superalgebras of classical type. These categories have properly stratified structures, and under this duality, the simple and tilting objects correspond. This equivalence is a generalization of the so-called super duality between certain parabolic BGG categories of Lie algebras and Lie superalgebras of classical type. Using this super duality of Whittaker modules, we establish an equivalence of certain categories of modules over the corresponding finite W-algebras and W-superalgebras. These categories are subcategories of W-(super)algebra modules that were originally introduced by Brundan-Goodwin-Kleshchev, and they contain all simple and standard objects. This is an ongoing joint work with Weiqiang Wang.

Kevin Coulembier (University of Sydney)

Prime ideals for Lie superalgebras

Abstract: We will talk about the prime spectrum in the universal enveloping algebra for quasi-reductive Lie superalgebras. In particular, we give the classification of prime ideals for Lie superalgebras of type I and talk about criteria for primeness of the enveloping algebra.

Maria Gorelik (Weizmann Institute)

On root systems of Kac-Moody superalgebras *(online)*

Abstract: In this talk I will describe the root bases and the sets of imaginary roots for Kac-Moody superalgebras. I will also discuss different manifestations of the Weyl group appearing in the context of Kac-Moody superalgebras. Joint work with Shay Kinamon Kerbis.

Victor Kac (MIT)

Representations of minimal W-algebras: unitarity and modular invariance

Abstract: Classification of unitary minimal quantum affine W-algebras and their (non-twisted and twisted) unitary representations will be explained, along with the computation of their characters. As an application, the denominator identities for all superconformal algebras follow. Modular invariance of modified characters will be discussed.

Jonathan Kujawa (University of Oklahoma)

Affine A-webs and their cyclotomic quotients

Abstract: Given an associative (super)algebra, A, we can construct diagrammatic categories depending on A that generalize those in the literature for the general linear Lie algebra and type Q Lie superalgebra. They also encode the representation theory of the RoCK blocks of the symmetric groups and, conjecturally, imaginary KLR algebras. This is joint work with Nick Davidson, Rob Muth, and Jieru Zhu.

Jae-Hoon Kwon (Seoul National University)

Crystal base for the quantum orthosymplectic superalgebras

Abstract: There is a family of irreducible highest weight modules over the quantum orthosymplectic superalgebra with crystal bases, called oscillator representations. Since no presentation is known for these modules, it is not easy to take a limit of their crystal bases. In this talk, we will talk about a crystal base of parabolic Verma modules and its compatibility with that of an irreducible oscillator representation. Then we take a limit of the crystal base of a parabolic Verma module to have a crystal base of the negative half of the quantum group. We also give a combinatorial description of the associated crystal embeddings. In particular, we have a monomial type basis of irreducible oscillator representations. This is a joint work with Uruno and Jang.

Sin-Myung Lee (Korea Institute for Advanced Study)

Towards quantum affine super duality *(short talk)*

Abstract: Super duality is an elegant approach in representation theory of classical Lie superalgebras, which explains at the category level some curious analogies between bosonic and fermionic constructions by introducing interpolating supersymmetric ones. In this talk, we explain how it can be lifted to representations of quantum affine (super)algebras using monoidal structures and R-matrices. This is based on an ongoing project with Jae-Hoon Kwon and Masato Okado.

Andrew Manion (NC State University)

Tensor products of higher representations of $gl(1|1)^+$

Abstract: I will give an overview of joint work with Raphael Rouquier constructing a tensor product operation for higher representations of Khovanov's categorification of $gl(1|1)^+$, along with its relationship to Heegaard Floer homology and recent developments.

Frantisek Marko (Pennsylvania State University)

Blocks of periplectic supergroups in positive characteristic *(short talk)*

Abstract: We describe blocks of periplectic supergroups $P(n)$ over the ground field of positive characteristic. Using the asymmetry of the root system of the periplectic supergroup, we establish a partial linkage of weights $\lambda \sim \mu$ in the case when the induced even module $H^0_{P_{ev}(n)}(\lambda)$ is irreducible. We utilize the combinatorial criteria for the irreducibility of the induced module $H^0_{GL(n)}(\lambda)$ given in the Ph.D. thesis of Jantzen to derive that up to the parity shift, there are two blocks of $P(n)$ represented by the weights $(0, \dots, 0)$ and $(0, \dots, 0, -1)$.

Ian Musson (University of Wisconsin-Milwaukee)

The Weyl groupoid in Type A, Young diagrams, Borel subalgebras and Cayley graphs

Abstract: Let $\mathtt{k}$ be an algebraically closed field of characteristic zero. Let $\overset{\circ}{\mathfrak{g}}$ be the Lie superalgebra $\mathfrak{sl}(n|m)$ and let $\mathfrak{T}_{iso}$ be the groupoid introduced by Sergeev and Veselov with base the set of odd roots of $\overset{\circ}{\mathfrak{g}}$. We show the Cayley graphs for three actions of $\mathfrak{T}_{iso}$ are isomorphic, These actions originate in quite different ways. The first arises from Young diagrams contained in a rectangle with n rows and m columns, the second from Borel subalgebras of the affinization $\widehat{L}(\overset{\circ}{\mathfrak{g}})$ of $\overset{\circ}{\mathfrak{g}}$ which are related by odd reflections. The third action comes from an action of $\mathfrak{T}_{iso}$ on $\mathtt{k}^{n|m}$ defined by Sergeev and Veselov motivated by deformed quantum Calogero-Moser problems.

Jonas Nehme (University of Bonn)

The periplectic Lie superalgebra, KLR algebras and categorification

Abstract: It is well-known that parabolic category $\mathcal{O}$ categorifies tensor powers of the natural representation for the general linear Lie algebra. In this setting, the action of the universal enveloping algebra is categorified via translation functors. In this talk, we want to consider a similar story for the periplectic Lie superalgebra $\mathfrak{p}(n)$. We will introduce a graded KLR-algebra of type P, whose level 1 cyclotomic quotient recovers the super Brauer category, i.e. the Schur–Weyl duality counterpart for $\mathfrak{p}(n)$, thus acting on the representation category via translation functors. We will show that this KLR-algebra of type P categorifies the quantum electric algebra, a quantization of the electric Lie algebra which appears in the study of electrical networks. We define an action of this quantum electric algebra on the Fock space by considering it as a coideal and show that this is categorified by the action of the KLR algebra of type P on the super Brauer category. (jt. with Catharina Stroppel)

Shifra Reif (Bar-Ilan University)

A Gruson–Serganova Character Type Formula for the Periplectic Lie superalgebra

Abstract: In 2019, a breakthrough in understanding the decomposition numbers for the periplectic Lie superalgebra was obtained by Balagovic, et al. We shall use this work to obtain finite character formulas for the periplectic Lie superalgebra. Joint work with Mee Seong Im.

Lev Rozansky (University of Chapel-Hill)

Quiver varieties for super-algebras $gl(m|n)$

Abstract: It is well-known that coherent sheaves over the special Nakajima quiver varieties categorify weight spaces in the tensor product of the fundamental representations of $gl(N)$. I will explain the string theory inspired modification of this construction that should lead to the categorification of the tensor product of some representations of $gl(m|n)$. This modification requires a reinterpretation of the Hamiltonian reduction used in Nakajima's construction as an intersection of Lagrangian subvarieties in a commuting variety (pairs of commuting matrices modulo conjugation). The super-algebra related varieties emerge after the Legendre transformation of some of these subvarieties. This is a joint work with R. Rimanyi and A. Oblomkov.

Siddharta Sahi (Rutgers University)

Supersymmetric Shimura operators and interpolation polynomials

Abstract: The Shimura operators are a certain distinguished basis for invariant differential operators on a Hermitian symmetric space. We consider the analogs of Shimura operators for the Hermitian symmetric superpair (g, k) where $g = gl(2p|2q)$ and $k = gl(p|q) \oplus gl(p|q)$ and we prove their Harish-Chandra images are specializations of certain BC-supersymmetric interpolation polynomials introduced by Sergeev–Veselov. The analogous statement for the non-super case was proved by Sahi-Zhang, who showed that the Harish-Chandra images of these operators are specializations of certain BC-symmetric interpolation polynomials defined by Okounkov, thereby answering a question of Shimura. This is joint work with Songhao Zhu, and the result was conjectured in earlier work by Zhu.

Hadi Salmasian (University of Ottawa)

Real forms, real representations, and the 10-fold way

Abstract: We give a complete description of finite dimensional irreducible real representations of real forms of basic classical Lie superalgebras by obtaining a correspondence between these modules and the modules of the complexification (modulo an involution obtained by conjugation). We also compute the endomorphism algebras of these real modules. This talk is based on joint work with Siddhartha Sahi and Vera Serganova.

Steffen Schmidt (Heidelberg University)

On the superdimension of unitarizable supermodules *(short talk)*

Abstract: We define a superdimension for any unitarizable supermodule over the complex special linear super Lie algebra, which bridges, in particular cases, the stream between representation theory and the superconformal index, appearing as an invariant in superconformal physical theories. The construction of the index is based on the formal dimension of discrete series representations and Weyl's dimension formula for the underlying real Lie group. We prove that the superdimension vanishes on simple unitarizable supermodules unless their infinitesimal character has maximal degree of atypicality, while also employing the Duflo–Serganova functor and a translation principle to compare the superdimension with the well-known finite-dimensional case.

Vera Serganova (University of California, Berkley)

Support variety for algebraic supergroups.

Abstract: We consider the stable category of representations of a quasireductive algebraic supergroup over C, i.e., supergroup with reductive underlying group. This is a triangulated tensor category and we address a problem of describing its Balmer spectrum. After a brief introduction to triangulated tensor geometry we formulate the conjecture in our situation and discuss two essential steps of its proof: projectivity detection and realizability. We present the proof of projectivity detection for all quasireductive supergroups and proof of realizability in some particular cases. We also discuss the cohomological support of Boe, Kujawa and Nakano. This is a joint work with J. Pevtsova and A. Sherman.

Alexander Sherman (University of Sydney)

Towards local representation theory for supergroups

Abstract: Local representation theory is a powerful approach to the modular representation theory of finite groups. It asserts that much can be understood about representation theory modulo p via restriction to (normalizers of) p-Sylow subgroups and arbitrary p-subgroups more generally. We will define analogues of p-groups, p-Sylow subgroups, and elementary abelian p-groups in the super setting. Sylows' theorems will be stated in the super context, and we will give several examples. Berezin volumes of homogeneous superspaces play a critical role, and we will explain this connection. Part of joint work with V. Serganova and D. Vaintrob.
