W3 Particle systems, nonlinear diffusions, and equilibration
(org. by G. Ben Arous, E. Carlen, F. Otto, K.T. Sturm, C. Villani)
November 12-16, 2007

Program:
All talks will take place in the HIM Lecture Hall in Poppelsdorfer Allee 45.

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Sebastian Andres

Title: Particle Approximation of the Wasserstein Diffusion

Abstract: In this talk a finite dimensional approximation of the recently constructed Wasserstein diffusion on the unit interval is presented. More precisely, the emperical measure process associated to a system of interacting, two-sided Bessel processes with dimension $0 < \delta < 1$ converges in distribution to the Wasserstein diffusion under the equilibrium fluctuation scaling. The passage to the limit is based on Mosco convergence of the associated Dirichlet forms in the generalized sense of Kuwae/Shioya. This is joint work with Max von Renesse.

Louis-Pierre Arguin

Title: The Structure of Quasi-Stationary Competing Particle Systems

Abstract: We study point processes on the real line whose configurations X can be ordered decreasingly and evolve through increments which are functions of correlated gaussian variables. The correlations are intrinsic to the points and given by a covariance matrix $Q = q_i j$. A probability measure on the pair (X, Q) is quasi-stationary if the joint law of the gaps of X and of Q is invariant under the evolution. A class of quasi-stationary processes is given by the Ruelle Cascades, which are based on hierarchally nested Poisson-Dirichlet processes. It was conjectured that these processes exhausted the class of robustly quasi-stationary laws. We prove a version of this conjecture when $q_i j$ assume only a finite number of values. The result is of relevance for mean-field spin glasses, where the evolution corresponds to the cavity dynamics, and where the hierarchal organization of the Gibbs measure was first

Fabio Martinelli

Title: Facilitated spin models

Abstract: We will review some recent advances in a rigorous description of reversible interacting particle systems with dynamical constraints and glassy behavior.

Jonathan Mattingly

Title: Spectral gaps in for degenerate SPDEs

Shin-ichi Ohta

Title: Gradient flows on Wasserstein spaces over compact Alexandrov spaces

Abstract: The Wasserstein space $(\mathcal{P}(X), d_2^W)$ is the space of Borel probability measures on a metric space X equipped with a suitably defined distance structure. Recently, it is turned out that there is a strong connection between the structures of $\mathcal{P}(X)$ and X. One of the most fascinating contributions is an approach to a synthetic lower Ricci curvature bound for general metric measure spaces due to Lott, von Renesse, Sturm and Villani. There the (semi-)convexity of the relative entropy is adopted as a substitution of the lower Ricci curvature bound.

In this talk, we treat the geometric structure of the Wasserstein space $\mathcal{P}(X)$ over a compact Alexandrov space X (a metric space with lower sectional curvature bound), and its application to constructing a gradient flow of a semi-convex function on $\mathcal{P}(X)$. We remark that a compact metric space X is an Alexandrov space of nonnegative curvature if and only if so is $\mathcal{P}(X)$. Roughly speaking, our results are the following:

proposed as an ansatz. This is joint work with M. Aizenman.

Thierry Bodineau

Title: Current large deviations for stochatic dynamics

Pietro Caputo

Title: On the approach to equilibrium for a polymer with adsorption and repulsion.

Abstract: We present some recent results on the decay to equilibrium of a Glauber dynamics for a pinning model. We derive several estimates on the relaxation time and mixing time of the process which illustrate the well known localization/delocalization phase transition from the dynamical point of view. Some open problems are also discussed. This is joint work with Fabio Martinelli and Fabio Lucio Toninelli.

Paolo Dai Pra

Title: Asymptotic behavior of nonsmooth multiplicative functionals of diffusion processes and related Hamilton-Jacobi-Bellman equations

Abstract: Joint work with Patrick Cattiaux, Université de Toulouse and Ecole Polytechnique, Sylvie Rölly Universität Potsdam
Consider the d-dimensional stochastic differential equation $dx_t = b(x_t)dt + dB_t$ and, assuming existence of solutions, the functional $S(T,x) := \log E_x \left[\exp\left(\int_0^T c(x_t)dt \right) \right. (0)$ where E_x is the expectation conditioned to $x_0 = x$. Under reasonable ergodicity conditions on the diffusion, one expects

1: $S(T,x) = lT + V(x) + o(1/T)$ where the pair (l,V) solves the ergodic HJB equation

2: $l = \frac{1}{2}\Delta V + b \cdot \nabla V + \frac{1}{2}|\nabla V|^2 + c.$

Facts 1 and 2 are well understood when $c(\cdot)$ is smooth and bounded; for $c(\cdot)$ smooth and with quadratic growth various results are available on equation 1, although a precise relation with the asymptotics in 1 has not been given. In this work we allow $c(\cdot)$ to be $discontinuous$ and quadratically growing. Under reasonable ergodicity conditions on the underlying diffusion we show that for $||$ sufficiently small the asymptotics in 1 exist, and that 2 is satisfied in a generalized viscosity sense. We used the method of cluster expansion, which is rather unusual in this context. The conditions for this expansion to converge translate in mixing conditions on the diffusion process, which seem to be of independent interest.

(1) Let X be a compact Alexandrov space of curvature bounded below. Then the Euclidean tangent cone exists at each point in $\mathcal{P}(X)$, although $\mathcal{P}(X)$ is not an Alexandrov space any more. It provides a kind of Riemannian structure on $\mathcal{P}(X)$, as was formally constructed by Otto on $\mathcal{P}(\mathbb{R}^n)$. The 2-uniform smoothness coming from Banach space theory plays an interesting role in the discussion.

(2) Let X be a compact Alexandrov space of curvature bounded below, and $f : \mathcal{P}(X) \longrightarrow (-\infty, \infty]$ be a semi-convex function. Then a complete gradient curve starts from any point $\mu \in \mathcal{P}(X)$ with $f(\mu) < \infty$. If X is of nonnegative curvature (this is only a technical assumption), then the gradient flow is unique and satisfies the contraction property. The proofs are much indebted to works of Perel'man-Petrunin, Lytchak and Ambrosio-Gigli-Savar\'e.

(3) If X is a compact Riemannian manifold of nonnegative sectional curvature (this is also a technical assumption) and m is its volume element, then the density function of the gradient flow in $\mathcal{P}(X)$ of the free energy $f(\mu) = \int_X \log(\mu/m)\, d\mu + \int_X V\, d\mu$ coincides with the solution of the associated linear Fokker-Planck equation $\partial\rho_t/\partial t = \Delta\rho_t + \mathrm{div}(\rho_t \cdot \mathrm{grad}V)$ on X. It extends a celebrated work due to Jordan-Kinderlehrer-Otto on Euclidean spaces.

Stefano Olla

Title: Macroscopic non-equilibrium behavior of chains of oscillators perturbed by a conservative noise.

Abstract: I will review some recent work on the macroscopic non-equilibrium behavior of the Hamiltonian dynamics of chains of oscillators perturbed by a conservative noise. The stochastic perturbation is given by hypoelliptic diffusions on the momentum of particles, such that total energy, and eventually total momentum, is conserved. I will discuss hydrodynamic kinetic and weak coupling limits for these models. These are joint works with Giada Basile, Cedric Bernardin, Milton Jara, Carlangelo Liverani and Herbert Spohn.

Robert Philipowski

Title: Approximation of the viscous porous medium equation by nonlinear stochastic differential equations

Abstract: We study a family of nonlinear stochastic differential equations indexed by a parameter > 0 and show that the distributions of the solutions converge for $\to 0$ to the solution of the viscous porous medium equation. Furthermore we explain how this result can be applied to the study of interacting particle systems.

Tadahisa Funaki

Title: Hydrodynamic limit for the $\nabla\phi$ interface model with a non-convex potential via two-scale approach

Abstract: We will discuss the hydrodynamic limit for the Ginzburg-Landau $\nabla\phi$ interface model for a certain class of non-convex interaction potentials, based on a two-scale approach recently introduced by Grunewald, Otto, Reznikoff and Villani combined with a recent work of Cotar, Deuschel and Mueller showing the strict convexity of the corresponding surface tension.

Arnaud Guillin

Title: Convergence to equilibrium via Lyapunov functional (and functional inequalities)

Martin Hairer

Title: Slow energy dissipation in chains of anharmonic oscillators

Abstract: We study the dynamic of a very simple mode of a chain of anharmonic oscillators with linear nearest-neighbour couplings. The first and the last oscillator furthermore interact with heat baths through friction and noise terms. If all oscillators in such a system are coupled to heat baths, it is known that under relatively weak coercivity assumptions, the system has a spectral gap (even discrete spectrum) and thus returns to equilibrium exponentially fast. It turns out that while it is still possible in certain cases to show the existence and uniqueness of an invariant measure for the general model, it returns to equilibrium much slower than one would at first expect. In particular, even in the simplest case of three oscillators, the spectrum of the system is no longer discrete when the potential of the oscillators is quartic and the spectral gap is destroyed when it grows even faster. This is due to the appearance of metastable states that can be described with high accuracy.

Dmitry Ioffe

Title: Ballistic Phase of Self-Interacting Random Walks

Abstract: I shall explain a unified approach to a study of ballistic phase for a large family of self-interacting random walks with a drift andself-interacting polymers with an external stretching force. The approach is based on a recent version of the Ornstein-Zernike theory. It leads to local limit results for various observables (e.g. displacement of the end-point or number of hits of a fixed finite pattern) on paths of n-step walks (polymers) on all possible deviation scales from CLT to LD. The class of models, which display ballistic phase in the "universality class" to be discussed in the talk, includes self-avoiding walks, Domb-Joyce model, random walks in an annealed potential, re-inforced polymers and This is a joint work with A. Figalli.

Fraydoun Rezakhanlou

Title: Moment Bounds for Smoluchowski Equation

Abstract: Smoluchowski's equation with diffusion is used to describe the evolution of cluster densities in a model of coagulating and fragmenting clusters. I discuss various bounds on moments of the solutions provided that the coagulation propensities and diffusion rates satisfy a suitable inequality. This inequality is needed to guarantee that no gelation occurs in finite time. As a consequence of the moment bounds, we infer the uniqueness of a weak solution and its conservation of mass.

Giuseppe Savare

Title: The Wasserstein gradient flow of the Entropy functional

Abstract: We will discuss some properties of the gradient flow of the logarighmic entropy functional in Wasserstein spaces. We will focus on the interplay between some geometric properties of the underlying metric (even in a non smooth case) and some feature of the Wasserstein approach, concerning the generation, the stability, and the linearity of the flow.

Vladas Sidoravicius

Title: DLA models and positive recurrence of Markov chains - what determines the shape?

Peter Topping:

Title: L-mass transportation and Ricci flow

Abstract:

I will describe how one can use new and old types of optimal transportation problems to investigate Ricci flows. No knowledge of Ricci flow will be assumed.

Maria Westdickenberg

Title: Two recent results on the logarithmic Sobolev inequality

Abstract: The logarithmic Sobolev inequality (LSI) is a powerful tool for studying convergence to equilibrium in spin systems. The Bakry-Emery criterion implies LSI in the case of a convex Hamiltonian. What can be said in the nonconvex case? We present two new sufficient conditions for LSI. The first is a Bakry-Emery-type criterion that requires only LSI (not convexity) for the single-site conditional measures. The second is a two-scale condition: An LSI on the microscopic scale (conditional measures) and an LSI on the macroscopic scale (marginal measure) are combined to prove a

weakely re-inforced random walks. Joint work with Yvan Velenik.

Tony Lelievre

Title: Convergence of an adaptive Monte Carlo algorithm by entropy methods

Rossana Marra

Title: Stability of the front under a Vlasov-Fokker-Planck dynamics

Abstract: We consider a kinetic model for a system of two species of particles interacting through a long range repulsive potential and a reservoir at given temperature, undergoing first order phase transition. The model is described by a set of two coupled Vlasov-Fokker-Plank equations. The front solution, which represents the transition profile between two coexisting phases, is a one-dimensional stationary solution on the real line with given asymptotic values at infinity. We prove the asymptotic stability of the front for small symmetric perturbations and we give also the rate of convergence.

global LSI. The talk includes joint work with Natalie Grunewald, Felix Otto, and Cédric Villani.

Michael Westdickenberg

Title: Optimal Transport for the System of Isentropic Euler Equations.

Abstract : We propose a variational time discretization for the multi-dimensional isentropic Euler equations and study its properties. The approximation is inspired by the interpretation of certain degenerate parabolic equations as gradient flows on the space of probability measures, and by Dafermos' entropy rate criterion: The total energy should be decreased at maximal rate. This is joint work with Wilfrid Gangbo.