Final Report for the Hausdorff Trimester Program on

Discrete Optimization

organized by

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The Hausdorff Trimester Program on Discrete Optimization was held towards the end of the COVID-19 pandemic, and was the first trimester program since the start of the pandemic to be held (primarily) in person, albeit with some substantial hybrid and virtual components. Conducting the program in these circumstances was undoubtedly a challenge for everyone involved, including participants, organizers and HIM staff. Especially the start of the program (where the Summer School and first workshop were primarily online) and the end of the program, where a resurgence of COVID led to cancellations and other disruptions, were impacted. Despite this, and thanks also to the unbounded enthusiasm of those participants who were able to attend and interact in person, as well as thanks to the dedicated HIM team, the program was a great success.

This trimester program could be viewed as following on from a previous trimester program on Combinatorial Optimization, held in 2015. However, while building on this previous program in some ways, our goal was also to bring in new and exciting directions, and to broaden the scope. This is visible in the four main "themes" of the trimester program, which also correspond to the four workshops held during the program. We will roughly divide our discussion of the outcomes of the program between these four themes. However, a further explicit aim of the program was to encourage interaction between communities where we identified the potential for beneficial exchanges. We believe that we were extremely successful in this regard. For instance, interactions between experts in tropical geometry and in the theory of optimization, as well as interactions between experts in approximation algorithms and in parametrized complexity, have led to some of the biggest successes coming out of the program. We will highlight such interactions below.

Organization

Summer school. The Summer School at the start of the program was held primarily online. While this was not our preference, it did have the benefit of making the school accessible to a larger number of students and other researchers. It was a great success, with over 150 participants (an estimate; the number of registrations to the summer school was much higher, nearly 500). The four speakers—Michał Pilipczuk, Aaron Sidford, Ngoc Mai Tran and Rico Zenklusen—each gave an excellent lecture series, covering a diverse set of topics related to one of the four main themes of the program.

Workshops. There were four formal workshops during the program, each involving additional organizers with strong expertise in the specific topics.

- *Tropical geometry and the geometry of linear programming*, organized by Xavier Allamigeon, Jesús De Loera, Laura Sanità and László Végh.
- *Continuous approaches to discrete optimization*, organized by Chandra Chekuri, Daniel Dadush, Yin Tat Lee and Stephen Wright.
- *Approximation and relaxation*, organized by Fabrizio Grandoni, Neil Olver, Laura Sanità and Jens Vygen.
- *Parametrized complexity and discrete optimization*, organized by Parinya Chalermsook, Friedrich Eisenbrand, Fedor Fomin and Jesper Nederlof.

Introductory talks. We had regular sessions throughout the program where new arrivals were asked to give a short introduction about their research interests, problems they were interested in working on, etc. These sessions were a great success, helping participants to make promising research connections early in their visit. We also had a regular **seminar series** throughout the program.

Online interactions. Given that a substantial number of invited participants were unable to attend due to the pandemic, or had to curtail their stay, we attempted to maintain the sense of community online as well. This was primarily via the use of Slack, a popular discussion platform (we note, however, that subsequent changes to the free version of this platform have made it less attractive for research discussions, and would strongly suggest Zulip as an open-source alternative). While online collaboration simply does not match the experience of in-person interaction, it was useful. For example, many of the open problems discussed in-person by participants were recorded to the online workspace, leading to further discussions.

Tropical geometry and the geometry of linear programming

The trimester program led to significant and surprising developments in the key directions outlined in the proposal, in particular, on furthering our understanding of strongly polynomial computability of LP, and on diameter bounds. This was enabled by interactions among researchers in different subfields. Two PhD students among the long-term participants, Bento Natura and Zhuan Khye Koh, have been working on these problems and obtained some key results of their theses [42, 47] during the trimester program.

Strongly polynomial solvability of LPs

A central open question in the context of linear programming is the existence of a strongly polynomial algorithm for LP; one of Smale's problems for the the twenty-first century mathematics [54]. Interior point methods (IPMs) have been the most successful method both in theory and in practice in recent decades, with major breakthroughs in improved weakly polynomial running times for general LP and particular problems such as network flows; we discuss some of these developments in the next section.

At the same time, layered least squares IPMs give some of the strongest results on strongly polynomially solvable subclasses. This class of IPMs was introduced by Vavasis and Ye [61]. In recent work by the trimester program participants Dadush, Huiberts, Natura, and Végh [21], these results were further strengthened, by studying a combinatorial 'circuit imabalance measure', and making use of matroid optimization techniques.

While these algorithms represent the state of the art for strongly polynomial solvability, a breakthrough 2018 result by Allamigeon, Benchimol, Gaubert and Joswing [3] showed that log-barrier IPMs *cannot* be strongly polynomial. The proof is based on tropical geometry, by interpreting the tropicalization of linear programming, and analyzing the tropical central path.

The trimester program brought together a group of researchers working on the two aspects of this question: Dadush, Huiberts, Natura, and Végh from the combinatorial IPM side, and Allamigeon, De Loera, Joswig, and Loho from the tropical lower bounds side. Discussions led to two significant developments on understanding the behaviour of IPMs. Firstly, the paper *No self-concordant barrier interior point method is strongly polynomial* by Allamigeon, Gaubert, and Vandame [5] (STOC 2022), strengthened the impossibility results to IPMs with any self-concordant barrier function. Secondly, Allamigeon, Dadush, Loho, Natura, and Végh started discussions in HIM on central path curvature measures and the best achievable running times of IPMs. This resulted in the FOCS 2022 paper *Interior point methods are not worse than Simplex* [4]. This complements the lower bounds in [3] and [5] by introducing a 'universal IPM' that is optimal in a strongly polynomial sense. The number of iterations is within a small strongly polynomial factor of any IPM with a self-concordant barrier function, and—similarly to the simplex algorithm—also admits a single exponential upper bound. This work introduces a powerful new tools that may be expected to lead to further developments in strongly polynomial analysis.

Diameter bounds for polyhedra

Another central question of linear programming is the *polynomial Hirsch conjecture*, i.e., polynomially bounding the diameter of a polytope in terms of the dimension and the number of facets. Besides the combinatorial diameter, defined by edge movements, there has also been significant work on the circuit diameter, that enables a natural, broader set of circuit directions. The trimester program and in particular the first workshop brought together a number of experts working on these bounds, including Black, Borgwards, Dadush, De Loera, Kafer, Sanità, Sanyal, Sukegawa, and Weltge.

Shadow simplex arguments were used in three papers partly developed at HIM. In the SoCG 2022 paper *Asymptotic bounds on the combinatorial diameter of random polytopes* [13], Bonnet, Huiberts, Dadush, Grupel, and Livshyts give upper and lower bounds on the combinatorial diameter of random spherical polytopes. Black's SODA 2023 paper *Small shadows of lattice polytopes* [8] gave the first polynomial bound on the monotone diameter of lattice polytopes. The paper was a finalist of the 2022 INFORMS George Nicholson Paper Competition. Lastly, the paper *On the Simplex method for 0/1 polytopes* [7] by Black, De Loera, Kafer and Sanità, show that the number of non-degenerate steps taken using some particular shadow pivot rules is linear in the dimension or in the number of variables of a linear program defined on a 0/1 polytope. The paper is currently under minor revision for the journal Mathematics of Operations Research.

In the context of circuit diameters, Dadush, Koh, Natura, and Végh noticed during discussions in HIM that the circuit imbalance measure used in the above mentioned work on LP can also be used to obtain strong bounds on circuit diameters. In the IPCO 2022 paper *On circuit diameter bounds via circuit imbalances* [23], they give circuit diameter bounds that are polynomial in the dimension and the logarithm of the circuit imbalance measure.

Tropical geometry and matroid theory

Discussions started during the trimester program led to a number of developments in tropical geometry and matroid theory. These include new structural results, as well as applications in machine learning and for graph divisors.

The long-term participant Loho initiated a number of projects with the short term participants Joswig, Brandenburg, Hertrich, and further co-authors. In *Generalized permutahedra and positive flag Dressians* (FPSAC 2023, IMRN 2023) [38], Joswig, Loho, Luber, and Olarte provide characterizations of generalized permutahedra. In *Tropical Positivity and Determinantal Varieties* [14], Brandenburg, Loho, and Sinn study signed tropicalizations of determinantal varieties. In *Signed tropical halfspaces and convexity* [45], Loho and Skomra develop a refined version of signed tropical convexity that lays the foundation for a further understanding of signed tropicalization of varieties and semi-algebraic sets.

In the ICLR 2023 paper *Lower bounds on the depth of integral ReLU neural networks via lattice polytopes* [31], Haase, Hertrich, and Loho analyze the expressibility of ReLU neural networks using the duality between neural networks and Newton polytopes via tropical geometry, and provide tight bounds on the number of hidden layers needed for the exact representation of certain functions, under the assumption that only integral weights can be used.

Husić, Koh, Loho, and Végh started working at HIM on the IPCO 2023 paper *On the correlation gap of matroids* [35]. The paper analyzes the correlation gap of weighted matroid rank functions parametrized by the rank and girth of the matroid. The correlation gap bounds the performance guarantee in a range of approximation and mechanism design settings.

The paper *On approximating the rank of graph divisors* (Discrete Mathematics, 2023) [6] by Bérczi, Hoang, and Tóthmérész presents computational hardness results in the context of graph divisors and tropical curves, and was developed following discussions between the first two authors at HIM.

Continuous approaches to discrete optimization

An important goal of semester, originally stated in the proposal, was to spur the development of improved algorithms for fundamental problems in combinatorial optimization using tools from continuous optimization. The program was tremendously successful from this perspective, yielding breakthrough advances for flow problems, fast approximation algorithms, submodular optimization and matrix discrepancy.

Minimum cost flow

The maximum flow asks for maximum amount of flow one can route on a capacitated graph between from a source to target node; the minimum cost flow problem asks for cheapest way, as measured by a cost function on the edges, to route desired flow demands through the graph. These are some of the most basic problems in combinatorial optimization, and fast algorithms to solve them problems have been intensively studied for more than a half century. The state of art in fast flow solving was improved by two overlapping groups of participants. Participants Jan van den Brand, Yang Liu and Aaron Sidford together with their co-authors developed an $\tilde{O}(m^{3/2-1/58}\log^2 U)$ [60], time algorithm for minimum cost flow on a graph with m edges and integral capacities bounded by U; this result was presented at STOC 2022. They developed faster dynamic data structure for maintaining electrical flows on graphs, which are the key component for speeding up the iterations of an interior point method for the flow linear program. This improved on the previous running time of $\tilde{O}(m^{3/2-1/328}\log U)$ due to Yu Gao, Yang Liu and Richard Peng [28] (presented by Yang Liu at the workshop), which was the first improvement for sparse graphs on the seminal $\tilde{O}(m^{3/2}\log U)$ time algorithm of Goldberg and Rao [30].

With this impressive technical improvement, the tantalizing question of whether all flow problems could be solved in essentially linear time remained open. In a recent breakthrough, long-term participant Rasmus Kyng, workshop participants Yang Liu and Maximilian Probst Gutenberg, and their co-authors gave an $m^{1+o(1)}\log U$ time algorithm [19]. This work received the best paper award at FOCS 2022, and was further highlighted in a recent article in Quanta [39]. During their time at Hausdorff, the authors worked on the simpler p-norm version of this result (where the capacity constraints are relaxed), which directly fed into the full version of their result. Fascinatingly, a key to their improvement was to work with an interior point method for which iteration corresponds to an improving *circuit augmentation*, a key concept explored throughout the trimester, for which the dynamic maintenance problem becomes more tractable.

While the last algorithm is essentially optimal, one drawback is that it is randomized; that is, the algorithm can make errors (albeit with small probability). An interesting and important direction is to understand when alternative deterministic algorithms exist achieving similar guarantees. Long term participant Sorrachai Yinchareonthawornchai and his co-author Thatchapol Saranurak examined this question for the vertex connectivity problem, which reduces to solving a polylogarithmic number of maximum flows [44] (this result was presented by Debmalya Panigrahi during the workshop). In their new work [53] appearing at FOCS 2022, they gave a deterministic $m^{1+o(1)}2^{O(c^2)}$ time algorithm for deciding whether the vertex connectivity is at most c (almost linear for $c = o(\sqrt{\log m})$.

Submodular Function Minimization and Matrix Discrepancy

Submodular function minimization is a fundamental problem in combinatorial optimization, and with many modern applications in machine learning and other fields. In work that began during the program, where 3 of the 4 authors were participants, Chakrabarty, Graur, Jiang and Sidford [16] give new lower bounds on the number of submodular function evaluations needed for any algorithm for this problem, giving the first superlinear lower bound. The paper by the long-term participant Haotian Jiang [37], concerned with a more general setting of convex optimization given a separation oracle, also benefited from discussions during the program.

An important question in discrepancy minimization is whether a fundamental theorem of Spencer regarding balancing vectors in the infinity norm [55] can be extended to balancing matrix in the operator norm, known as the *matrix Spencer conjecture* [46]. During the workshop, long-term participants Daniel Dadush and Haotian Jiang together with their co-author Victor Reis developed a new convex geometric framework for attacking this question, by relating a related covering problem to the existence of fast first-order methods over the spectrahedron. In work published in the STOC 2022 conference [22], they used this framework to give new bounds on the Matrix Spencer conjecture and provide a pathway to proving the full conjecture.

Approximation and relaxation

Breakthrough results on TSP [56, 57, 58] before the trimester program made use of dynamic programming in a novel way, and in a way that evokes parameterized algorithms. This was discussed in the original proposal; we suggested that more interaction between these groups of researchers could be fruitful.

This indeed turned out to be the case. Vera Traub, an expert in approximation algorithms (and recent winner of the 2023 Maryam Mirzakhani New Frontiers Prize), highlighted interactions during the program with Michał Pilipczuk, an expert in parameterized algorithms and structural graph theory. These discussions were very useful in her breakthrough [59] along with Rico Zenklusen (another participant) on the Weighted Connectivity Augmentation problem, which asks for the cheapest way of enhancing a starting network to increase its connectivity. Before their work, the best known approximation factor for the problem was 2, which had not improved since the 90s. Recent progress on the tree augmentation problem, a special case, has received a lot of attention in the community (and was discussed in detail by Rico Zenklusen as part of the summer school at the start of the program). Their result is a $(1.5+\epsilon)$-approximation, published at the STOC 2023 conference.

A number of other interesting results related to approximation algorithms for network optimization problems were established by participants of this program. In work to appear at FOCS 2023, Nathan Klein and Neil Olver [41] make progress on the notorious *Thin Tree Conjecture* of Goddyn. This conjecture has received attention both as a basic question in graph theory, and due to its implications for the asymmetric traveling salesman problem and other problems in combinatorial optimization. Their new result shows how to find a spanning tree in a graph which is well-behaved on a given laminar family of cuts. Dylan Hyatt-Denesik, Afrouz Jabal Ameli, and Laura Sanità [36] developed improved approximation results for some node-connectivity augmentation problem, which will be presented at ICALP 2023. Both these two works were initiated fully during the program.

While the focus in approximation has typically been to reduce the approximation factor, a recent trend has been to explore whether approximation algorithms can be made extremely fast. Long-term participants Sorrachai Yingchareonthawornchai and Parinya Chalermsook together with co-authors examined this question for the minimum cost k-edge connected spanning subgraph problem (k-ECSS). In work that appeared in ICALP 2022 [33], they designed an $\tilde{O}(m/\varepsilon^2)$ time algorithm that returns an $(1+\varepsilon)$-approximate solution to the linear programming (LP) relaxation of k-ECSS problem, by combining the multiplicative weight update method together with fast datastructures. By a suitable rounding procedure, this can be converted into a integral k-ECSS $(2+\varepsilon)$ approximate solution in almost the same time.

Our program was successful in fostering new research collaborations. As an example of this, we mention that recent results on a special case of the sparse hitting set problem [9] have their roots in an open question asked by Andreas Feldmann during the program, and benefited from subsequent discussions with other participants. Another connection was made between Marco

Cauduro, who was working on independence and hitting set problems for axis-parallel rectangles with Andras Sebő, and a group of long-term participants who were working on independence problems for axis-parallel segments. The resulting preprint [15] gives an optimal worst-case bound on the independence number of a particular class of graphs: Namely, intersection graphs of axis-parallel segments in the plane where no three segments intersect at a single point. This work has been accepted to the Journal of Combinatorial Geometry.

Our program served also the purpose of consolidating long-lasting research collaborations. Ahmad Abdi and Gérard Cornuéjols, who collaborated extensively during the previous Trimester program on Combinatorial Optimization, continued their successful collaboration in this program as well. Very exciting progress on a famous conjecture of Woodall [2] is under review for publication; the most technical and sophisticated result of the paper was proved during the period that both Abdi and Cornuéjols were attending the program. A paper on the new topic of dyadic integer programming has also been published [1].

In work largely done during the program, Hunkenschröder, Pokutta and Weismantel [34] consider the minimization of a convex function of a particular form over the 0/1 hypercube, an NP-hard problem. They give faster (non-polynomial) running times for this problem, using an approach based on a new proximity result, that relates the minimizer to the optimum of the associated convex relaxation.

A number of results on scheduling, a long-standing and well-developed area, were impacted by the program. Some of these were connected to parametrized algorithms, and will be discussed in the next section. There were also results by participant Jannik Matuschke and his co-authors, who address some generalized malleable scheduling where jobs can be processed simultaneously on multiple machines: the jobs processing times are determined by the joint processing speed of the allocated machines, which follow some different discrete concavity assumptions. The authors develop new approximation algorithms, and also perform computational experiments to evaluate in practice the tightness of their theoretical worst-case guarantees. The paper [27] was presented at IPCO 2022, while [26] is currently under minor revision in the journal Operations Research.

Participants Nicole Megow and Franziska Eberle and co-authors [25] study a quite general configuration balancing problem with stochastic requests, which captures several natural resource allocation problems where the goal is to find a fair allocation of the resourses among a set of competing requests. They consider both the offline and online setting, and give approximation and competitive algorithms. Fair allocations are also in the focus of the work of Sanità and co-authos [29], though in the contex of network bargaining games and cooperative matching games, which are popular game theory problems involving the structure of matchings in graphs. They develop exact and approximation algorithms to ensure that instances of these games admit stable solutions (fair allocations). The latter two works were both presented at IPCO 2023.

Continuing with the area of algorithmic game theory, we mention recent progress on understanding equilibria in flow-over-time traffic models by participants Neil Olver and Laura Vargas Koch, along with Leon Sering [52]. The paper will appear at FOCS 2023, and builds on their previous work [51] which was presented during the workshop. We also mention work by Hommelsheim, Megow and Peiss on a new model of recoverable robust optimization [32].

Finally, participants Chandrasekaran, Fiorini, Weltge and co-authors initiated a polyhedral study of the feedback vertex set problem during the program, which evolved into the recent preprint [18]. Here they prove the existence of an IP formulation for the problem whose LP-relaxation can be solved in polynomial time and has an integrality gap bounded by 2, answering an open question in the literature.

Parametrization and discrete optimization

A prominent theme within the direction "Parametrization and discrete optimization" of the program was to seek connections with approximation algorithms. There have been several successes in this respect. Some of this has already been mentioned [59, 15]. Another direction initiated during the program by long-term participants Parinya Chalermsook, Fedor Fomin, Tuukka Korhonen and Jesper Nederlof was to investigate how algorithmically useful the *treewidth* parameter, commonly studied in parametrized complexity, is for designing approximation algorithms. This resulted in a paper to appear at ESA 2023 [17].

The project originated in the open problems session and was advanced by Jana Cslovjecsek, Michał Pilipczuk, and Karol Węgrzycki. The collaborative effort culminated successfully within the same semester. The manuscript is currently under peer-review.

Jana Cslovjecsek, Michał Pilipczuk and Karol Wegrzycki have submitted their preprint on the parameterized approximation for maximum weight independent set in a certain class of geometric intersection graphs [20]. The project originated in an open problems session during the program and the results were obtained entirely within the same semester. The manuscript is currently under peer-review. Jesper Nederlof, Michał Pilipczuk and Karol Wegrzycki thoroughly investigated the space usage of Baker's Efficient Approximation Schemes for hard optimization problems on planar graphs, and obtained, as an unexpected by-product, a new structural insight on planar graphs [48].

Another direction was to apply the paradigm of parametrized complexity in the *scheduling* application domain. Even far before the start of the program, there has already been some great progress on the study of parametrized complexity of scheduling problem using discrete optimization techniques such as fast algorithms for (integer) linear programming with few variables, or block-structured integer programming. Since not all researchers were familiar with these techniques due to their interdisciplinary nature, this received a lot of attention. Kim-Manuel Klein, Adam Polak and Lars Rohwedder did research during the program on pseudo-polynomial time algorithms for a scheduling problem that resulted in a paper that appeared at SODA 2023 [40]. In their work, triangular structured ILPs play a crucial role. Additionally, there are still active ongoing collaborations (such as one involving participants from Warsaw University and EPFL) on block-structured ILPs.

Somewhat orthogonally to this, Jesper Nederlof, Céline M. F. Swennenhuis and Karol Wegrzycki spent a considerable amount of their time during the program on a scheduling problem with a bounded unrelated machines and jobs with unit processing times (known as $P3 \,|\, \text{prec}, p_j = 1 |\, C_{\max}$). It is an infamous open question (in fact, one of the few remaining ones from the standard textbook on NP-completeness by Garey and Johson) as to whether this problem is NP-complete, or solvable in polynomial time. The currently best approximation algorithms, initiated by a breakthrough result by Thomas Rothvoß and Elaine Levey [43], heavily rely on convex programming methods[1]. The work of the authors relies more on ideas from parametrized algorithms, and their work that started during the trimester program eventually led to faster algorithms for the problem. Their initial results are partly available in [50], and in a more recent unpublished manuscript [49] they even manage to get an algorithm that runs in time sub-exponential in the number of jobs.

The program, and especially the last few weeks (from the workshop dedicated to parametrization and discrete optimization onwards) also conceived some projects more centered in parame-

[1] A recent exception in this research line that does not rely on such optimization methods is [24].

terized complexity itself. An example of such work is the theory-building of new parameterized complexity classes; this effort greatly benefited from interactions during the program. Outcomes from this effort have already been published in ESA 2022 [10] and IPEC 2022 [11], with a follow-up work in ICALP 2023 [12].

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