

Report on the Hausdorff Trimester Program

Representation Theory, Complex Analysis and Integral Geometry

June - August 2007

Organizers: Simon Gindikin and Bernhard Krötz

Integral Geometry has its origins in the 19th century where one studies integral transforms of geometric nature. An important later example is the Radon transform, which starting with the inversion formula led to remarkable explicit formulas. In trying to generalize this explicit analysis one was looking for more general geometric structures where something like this can be done. As often in mathematics large symmetry is a great help and the most symmetric objects are Lie groups, here complex semi simple Lie groups, and related homogeneous spaces. This way representation theory entered in a very natural way and plays a central role in the modern development.

The aim of the program was to bring experts from the different mathematical areas which are involved into the subject together to stimulate discussions and new research. To educate the younger people (and not only the younger) several lecture series by outstanding mathematicians were organized.

The program was originally planned at the Max Planck Institute on a smaller scale, which in particular meant that senior mathematicians dominated. Some of the leaders of the field had agreed to take part, amongst them besides the organizers were Joseph Bernstein, Michel Duflo, Roger Howe, Eric Opdam, Wilfried Schmid, and Robert Stanton. The foundation of the Hausdorff Institute allowed to enlarge the program significantly by mainly inviting additional young researchers of very high level. Here are some of the names of young scientists, who might become future leaders: Avram Aizenbud, Dmitry Gourevitch, Omer Offen, Eitan Sayag. The atmosphere at the program was very lively and the activities ranged from a series of five minicourses over formal and informal seminars to a conference.

Several interesting papers emerged from the program. Two highlights should be mentioned. The first is a cooperation between two participants and Werner Müller from Mathematisches Institut of Bonn University. Tobias

Finis and Erez Lapid together with Werner Müller could prove the absolute convergence of Arthur's trace formula. The other cooperation lead to the paper: "Multiplicity one Theorems" by Aizenbud, Gourevitch, Rallis and Schiffmann, which appeared 2010 in the Annals of Mathematics. Here is the abstract of the paper:

"In the local, characteristic 0, non-archimedean case, we consider distributions on $GL(n + 1)$ which are invariant under conjugation by $GL(n)$. We prove that such distributions are invariant by transposition. This implies multiplicity at most one for restrictions from $GL(n + 1)$ to $GL(n)$. Similar Theorems are obtained for orthogonal or unitary groups."

It is a pleasure to quote from the acknowledgments in this paper:

"The first two authors worked on this project while participating in the program Representation theory, complex analysis and integral geometry of the Hausdorff Institute of Mathematics (HIM) at Bonn joint with Max Planck Institute for Mathematics. They wish to thank the organizers of the activity and the director of HIM for inspiring environment and perfect working conditions."

The program was a perfect start for the new Institute and demonstrated that involving mathematicians from the Max Planck Institute and Bonn University played an important role.