Report of the Hausdorff Trimester Program

The Interplay between High-Dimensional Geometry and Probability

Period of stay: January - April 2021

Organizers

Ronen Eldan (Rechovot) Assaf Naor (Princeton) Matthias Reitzner (Osnabrück) Christoph Thäle (Bochum) Elisabeth M. Werner (Cleveland)



Aims and Restrictions

The last ten years have seen a merger of ideas and techniques from analysis, geometry and probability and have led to breakthrough results. It was the goal of the trimester program at the Hausdorff Institute for Mathematics (HIM) to bring together senior and junior researchers in these areas for mutual exchange of ideas and methods, to intensify the already existing ties and to stimulate substantial progress at the crossroads of these disciplines.

Due to the situation of the COVID19 pandemic in the beginning of the year 2021, most activities of the program took place virtually. Many discussions that had started during the official program period, continued in smaller discussion groups on Zoom. Again, due to Covid, only a small group of (in total) 12 participants was able to visit the Hausdorff Institute in Bonn in person.

Research Topics

The research program of the trimester program was devoted to the following topics and their inter-connections:

- asymptotic geometric analysis and geometric functional analysis,
- high-dimensional convex geometry,

- geometry of high-dimensional measures and isoperimetric inequalities,
- stochastic geometry and random graph theory,
- random matrix theory,
- limit theorems and deviation inequalities.

Organization of the Program

The TP started with a introductory winter school **The Interplay between High-Dimensional Geometry and Probability**. The aim of this winter school was to introduce junior researchers to the different angles of the program on high dimensions. The main focus was to highlight the strong connections between probability theory on the one hand and geometry on the other hand. We had the following four series of lectures:

- Radoslaw Adamczak (Warsaw): Functional inequalities and concentration of measure,
- Boaz Klartag (Weizman Institute of Science): On Yuansi Chen's work on the KLS conjecture,
- Joe Neeman (Austin): Gaussian isoperimetry and related topics,
- Giovanni Peccati (Luxembourg): Some applications of variational techniques in stochastic geometry.

Additional topics were covered by P. Diaconis, M. Ludwig, and T. Tkocz.

An essential part of the program were two one-week workshops, where junior and senior scientists reported on their resent research. The organizers tried to select the speakers in such a way that the fruitful interplay between probabilistic and geometric aspects on high dimensional objects was always visible. The first workshop **High dimensional spatial random systems** (organized by Apostolos Giannopoulos, Friedrich Götze, Matthias Reitzner and Christoph Thäle) concentrated on results about high dimensional geometric and probabilistic limit theorems. Three series of lectures were given by

- Omer Bobrowski (Technion): Random simplicial complexes,
- Zakhar Kabluchko (Münster): Random polytopes,

• Joscha Prochno (Graz): The large deviations approach to high-dimensional convex bodies.

Further presentations were given by Francois Baccelli, Erik Broman, Günter Last, Ilya Molchanov, Tobias Müller, Eliza O'Reilly, Peter Pivovarov, Ngoc Mai Tran and D. Yogeshwaran.

The second workshop **High dimensional measures: geometric and probabilistic facets** (organized by Ronen Eldan, Alexander Litvak, Assaf Naor and Elisabeth M. Werner) was dedicated to probabilistic properties of high dimensional convex geometry. Speakers were Emanuel Milman, Olivier Guédon, Han Huang, Dongmeng Xi, Kateryna Tatarko, Franz Schuster, Andreas Bernig, Bo'az Klartag, Yuansi Chen, Stanislaw Szarek, Stanislav Nagy, Matthieu Fradelizi, Liran Rotem, Dmitry Ryabogin, Konstantin Tikhomirov, Pierre Youssef, Galyna Livshyts, Santosh Vempala, Alina Stancu, Vladyslav Yaskin, Dmitry Zaporozhets, Mark Meckes, Grigorios Paouris and Nike Sun.

In addition, we had a weekley **TP Zoom seminar series** with presentations given by more prominent scientists working at the interface between probability and high dimensional geometry: Kavita Ramanan, Shiri Artstein-Avidan, Keith Ball, Sergey Bobkov, Masha Gordina, Alexander Litvak, Daniel Hug, Alexander Kolesnikov, Andrea Colesanti and Ramon van Handel. Another vital component of the trimester program were **pre-recorded 10 min talks**, where especially younger participants were able to present their research work in a condensed form. The videos were collected on the TP's webpage during the whole program and there was an organized live discussions (via Zoom) for every single talk. All participants were invited to meet the speakers for live discussions of their contributions.

Finally, the program had four virtual problem groups in which participants were discussing and working on open problems.

- Problem Group 1: Random polytopes in spherical geometries,
- Problem Group 2: Towards the Aaronson-Ambainis conjecture,
- Problem Group 3: Polynomial Hirsch Conjecture for Random Polytopes,
- Problem Group 4: Convex Geometry and Depth.

All virtual lectures and presentations were recorded and uploaded on the HIM-channel on youtube. These recordings have reached a broad audience

as all have received several hundreds (sometimes even more than 1000) clicks. Also, all virtual events were very well visited, for example during the virtual winter school we had over 180 participants online.