Junior Trimester Program "Randomness, PDEs and Nonlinear Fluctuations"

02.09.2019–19.12.2019

Group 4 *Stochastics, roughness and fluctuation distribution.*

Members   Antoine Hocquet, Khoa Lê, Siragan Gailus, Fanhui Xu.

The junior Trimester Program was a unique occasion for the members of our group to initiate scientific discussions and collaborations.

We are particularly thankful to HIM administration for their constant support concerning practical issues.

Workshops

HIM gave us the possibility to organize 2 workshops:

- *Harmonic Analysis and rough paths (Nov. 18-19).* This workshop was initiated by A. Hocquet, P. Friz, K. Lê and P. Zorin Kranich (U Bonn). The main insight that motivated this workshop is the observation that Harmonic Analysis and rough paths can be regarded as 2 different ways to decompose a given signal (path). An objective was to combine the skills of two main institutes: TU Berlin (Rough paths: represented by the expertise of P. Friz) and Bonn University (Harmonic Analysis: represented by the expertise of C. Thiele), in order to highlight intersections between the 2 disciplines.

- *Problems of roughness, geometry and random fluctuations (Dec- 9-12).* This workshop was an attempt to gather members of different communities of probability theory around questions of geometry and stochastic analysis (including rough paths).

  In our opinion, this event – which gathered various experts such as M. Röckner, X.M. Li, H. Oberhauser, C. Litterer, G. Iyer, K-T.Sturm, P. Möters, L. Coutin and A. Deya (among others) – was a remarkable success. Stochastic PDE, geometry, and rough path theory which used to develop independently were brought together. Many discussions in the disciplinary area, such as highly oscillating nonlinear systems on manifolds, occurred during the coffee break for scientific discussions.

Scientific highlights

During the stay, Fanhui Xu worked on the well-posedness problem of the stochastic Navier-Stokes equation with multiplicative white noise. This collaboration with Igor Kukavica and Mohammed Ziane has been published in the journal Stochastics and Partial Differential Equations: Analysis and Computations [7]. They proved that the solution globally exists in

L^p spaces with large probability if $p > 5$ and if the initial datum is small. This conclusion resembles earlier deterministic results of Tosio Kato.

Fanhui Xu also had the opportunity to discuss with Sonja Cox after she delivered her talk. That mathematical discussion led to a later collaboration with Pawell Przybyllowicz and Michaela Szölgyenyi on SDE with discontinuous drift and finite many jump activities. This work has been published in the journal Statistics and Probability Letters [9]

Some ideas in the preprint [3] by P. Friz, A. Hocquet K. Lê were born during discussions in HIM. Notably, a discussion between K. Lê and M. Gerenscer was at the origin of the "stochastic Davie-Gronwall Lemma", as stated in [3].

Parallel to that, K. Lê started some preliminaries calculation for the preprint [8].

K. Lê also established contacts with Mate Gerenscer and Konstantinos Dareiotis. The preprint [2] was written later on, but the contact between the authors was made in HIM.

During the workshop "Stochastic Fluid Dynamics" (Nov. 11-15), A. Hocquet, M. Romito and L. Tolomeo started a project on pathwise regularisation by noise.

A. Hocquet and his guest A. Vogler (TUB) have been working on continuity, cubature approximations, neural network mckean vlasov stochastic optimal control. The discussions led to the publication [5]. An ungoing work including the participation of group member S. Gailus (along with their guest A. Vogler) focusing on cubature aspects is nearing completion [6]. Another highlight in this direction of research is the informal lecture on stochastic control by Laurent Pfeiffer (guest, INRIA - Polytechnique).

Another achievement occuring during the JTP was the collaboration with guest Emanuela Gussetti (U Bielefeld), which led to a preprint on large deviations for the Landau-Lifshitz-Gilbert equation in 1D [4]. It should be noted that E. Gusetti was a speaker during the second workshop of our group (Dec. 9-12).

S. Gailus also had some discussions with Khoa about ergodic multiscale homogenization with fractional Brownian motions. This preliminary step led to a publication on statistics for standard and fractional multiscale stochastic dynamical systems [1].

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Randomness, PDEs and Nonlinear Fluctuations

Hausdorff Institute Bonn, 02.09.2019–19.12.2019

March 25, 2021

Group: Frontiers in Singular Stochastic PDEs

Research areas: Stochastic Analysis, stochastic PDEs, statistical mechanics, interacting particle systems

Group members: Yvain Bruned, Giuseppe Cannizzaro, Ajay Chandra, Dirk Erhard, Hao Shen, Leonardo Tolomeo, Baris Evren Ugurcan, Weijun Xu, Willem van Zuijlen

Group leader: Giuseppe Cannizzaro

The members of the group "Frontiers in Singular Stochastic PDEs" participated to the Junior Trimester programme "Randomness, PDEs and Nonlinear Fluctuations" in the fall of 2019. The vibrant and exciting atmosphere enjoyed at the HIM was a true source of stimulus for all of us. The synergy with all the other groups was amazing and greatly contributed to the success of the programme.

Research-wise, our goal as a group was to investigate cutting-edge topics in and around singular SPDEs. Not only we aimed at gaining a deeper understanding and make advances in their analysis but also explore the ramified applications they have to algebra, statistical mechanics, functional and stochastic analysis.

In the inspiring atmosphere of HIM, our group obtained important results in: universality and properties of the Φ_3^4 equation, two-dimensional Yang-Mills measure, directed edge reinforced random walks, directed polymers and universality and properties of the KPZ equation, universality classes for random interface models beyond KPZ and EW, regularisation by noise phenomena, Schrödinger and focusing equations, quasilinear singular SPDEs, the parabolic Anderson model and the connected Anderson Hamiltonian, singular SPDEs in the critical dimension and in particular the Anisotropic KPZ equation.

We would like to express our deepest gratitude to the HIM for giving us the possibility of taking part to the programme. This unique chance to interact with mathematicians from many different research areas was truly inspirational for our personal and career paths and very much appreciated.

Summer School and Workshop

As part of the activities we organised as a group at HIM, there are the week-long Summer School "New Frontiers in Singular SPDEs and Scaling Limits", and the Workshop "Singular SPDEs and Related Topics", the first taking place at the end of September, the second at the end of October.

- The Summer school had 4 main speakers, Tadahiro Oh, Felix Otto, Fabio Toninelli, Nikolaos Zygouras. With about 70 participants, mostly students and postdocs, the summer school was very well-received, both for the topics covered by the lecturers and the quality of their presentations. The participants had the opportunity to present their own work in a poster session which was well-attended.

- The Workshop saw the participation of the most prominent mathematicians from all over the world working in the field of stochastic PDEs and related topics. The extremely high quality of the speakers and their talks, and the breadth of the meeting was further enhanced by the Plücker lecture by F. den Hollander and a plenary talk by the Fields medallist M. Hairer.

Visitors

During the programme, we had several visitors with whom all members of the group had the possibility to interact and learn from. The scope of the programme was significantly enhanced by their seminars and their presence, both triggering novel collaborations and allowing interesting discussions. These include

- Tadahiro Oh, Felix Otto, Fabio Toninelli, Nikolaos Zygouras, who gave lectures during the Summer School:

- Cyril Labbé, Tom Klose, Tommaso Rosati, Philipp Schönbauer, Huanyu Yang

Seminars and Lectures

During the semester, together with the other groups, we ran several seminars and lecture series with the aim of fostering the collaborations among participants and making each other aware of our own research topics. More specifically, we organised

- Informal Seminar ran weekly, in which participants presented their own works,

- Invited Seminars, in which external speakers (including visitors) were invited to HIM to give a talk (including M. Gubinelli, P. Ferrari, F. De Vecchi, N. Barashkov, I. Zachuber...)

- Lecture Series, W. van Zuijlen held a minicourse on Besov Spaces.

Research outcomes

In what follows, there is a list of research outcomes including collaborations, projects, work in progress and preprints or publications. Everything stated below was either carried out or initiated at the Hausdorff Institute by the members of our group, participants to the programme (both in **bold**) and their visitors (in *italic*).

- **D. Erhard** and **W. Xu** worked on the weak universality of the Φ^4_3 equation from a general microscopic smoothing mechanism, which resulted in the two outcomes [EX20b, EX20a].
- **D. Erhard**, T. Franco and *G. Reis* worked on directed edge reinforced random walk, a random walk that keeps track of the past and gives a bias towards directed edges previously crossed. This resulted in the preprint [EFR20].
- **A. Chandra**, I. Chevyrev, *M. Hairer* and **H. Shen** worked on the $2D$ Yang-Mills measure. They defined natural state space and Markov process associated to the stochastic Yang-Mills heat flow in two dimensions and resulted in the preprint [CCHS20].
- **H. Shen**, J. Song, R. Sun and L. Xu worked on a directed polymer model in dimension $1+1$, where the disorder is given by the occupation field of a Poisson system of independent random walks on $\mathbb{Z}$ and they proved that, the continuum and weak disorder limit, the latter converges to the Stochastic Heat equation. This resulted in the preprint [SSSX20].
- **A. Chandra**, **T. Gunaratnam** and *H. Weber*, worked on phase transitions for Φ^4_3. They established a surface order large deviation estimate for the magnetisation of low temperature Φ^4_3 and their outcomes can be found in the preprint [CGW20].
- **A. Hocquet**, *M. Romito* and **L. Tolomeo** started an on-going project about the links between various notion of path-wise irregularity and the phenomenon of regularisation by noise.
- **L. Tolomeo** and *H. Weber* continued their collaboration on the limiting properties of the Gibbs measure for the focusing nonlinear Schrödinger equation posed on $\mathbb{T}$, when the size of the torus grows to infinity.
- *T. Oh*, M. Okamoto, K. Seong and **L. Tolomeo** worked on the construction of Gibbs-type measures for focusing equations, and the study of the associated dynamics for wave equations, which resulted in the preprints [OST20, OOT20].
- **Y. Bruned** and **M. Gerencser** started an on-going cooperation on quasilinear SPDEs.
- **Y. Bruned** and *L. Zambotti* discussed about Zambotti's work on the geometry of the space of branched Rough Paths, which lead to the publication [Bru20].
- W. König, *N. Perkowski* and **W. van Zuijlen** worked on the Parabolic Anderson model in 2 dimension with white noise potential and established long-time asymptotics, resulting in the preprint [KPvZ20].

- **W. van Zuijlen** completed his work with K. Chouk on the asympotics of the eigenvalues of the Anderson Hamiltonian with white noise potential in two dimensions [CvZ21].
- **G. Cannizzaro**, *C. Labbé* and **W. van Zuijlen** started an ongoing project on the fluctuations of the eigenvalues of the Anderson Hamiltonian for correlated potentials.
- **H. Altman**, *T. Rosati* and **W. van Zuijlen** discussed about synchronisation of the KPZ equation.
- **G. Cannizzaro** and *M. Hairer* continued their work on a new universality class for random interface models in $(1 + 1)$-dimensions. They constructed the universal process describing the universal fluctuations of models in this class, the Brownian Castle and showed its universality. This resulted in two works [CH21b, CH21a].
- **G. Cannizzaro**, **D. Erhard** and *F. Toninelli* worked on the Anisotropic KPZ equation, a singular SPDE at the critical dimension, for which they determined the large-scale behaviour of the correlation function and showed that it is logarithmically superdiffusive. This resulted in the work [CET20].

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JTP Randomness, PDEs and Nonlinear Fluctuations

Group SPDEs and Fluid-dynamics – Final report

Group members: Luisa Andreis, Luigi Amedeo Bianchi, Michele Coghi,
Mario Maurelli, Alden Waters, Margherita Zanella

We had the opportunity of being part of the Junior Trimester Program on *Randomness, PDEs and Nonlinear Fluctuations* from September to December 2019. This period allowed us to focus on our research without the interruptions and distractions we sometimes face in our institutions. Moreover it was a great opportunity for us to be all together in the same place for a long period of time and thus be able to discuss and work on ideas on a longer time span than the one workshop and conferences allow. It was a very satisfying and fruitful period of time, and we are extremely grateful to the Hausdorff Institute for accepting us in the program. In this brief report we will try to give a rough picture of our trimester in Bonn.

The highlight of our group's activity was the Workshop *Stochastic Fluid Dynamics* that took place on November 11-15, 2019. The aim of the meeting was to bring together people interested in Stochastic fluid dynamics, intended in a broad sense. There were 48 registered participants and 20 speakers, from both the PDE and SPDE areas of interest, Thanks to HIM, we were able to spread the talks all over the week and leave substantial time every day for scientific discussion. This proved to be a smart decision, as it led to several deep and fruitful discussion between all participants and in particular members of the group.

We also had two mini-courses: one by Zdzisław Brzeźniak on *Stochastic Strichartz estimates* and one by Yves van Gennip on *Variational methods on graphs*.

During the Trimester, our group had a few guests: Zdzisław Brzeźniak, Gianmarco Bet, Wun Sun, Francesco Grotto, Nimit Rana, Jakub Slavik, Yves van Gennip, and Oleg Butkowski.

There were also many inter-group activities we all took benefit from: the joint biweekly seminar, the workshops organized by other groups and the mini-course on Besov Spaces by Willem Van Zuijlen.

Some of us are particularly thankful for the opportunity to participate in the Seminar *Proposal Writing – Key Factors for Success!* by Dr. Wilma Simoleit organized by the HCM Bonn. We really appreciated that he Hausdorff Center extended invitations to internal events also to participants of the Trimester.

We are thankful to the HIM administration for the support, which solved quickly the few small practical issues we had and allowed us to focus only on research.

Scientific outcome

The lively environment at HIM enhanced mutual interactions, in particular for Luisa Andreis exchanges of ideas on fluctuations and large deviations for interacting systems were extremely useful in working on an existing project which led to the paper [1], which is about to be submitted.

Luisa Andreis and Wun Sun, who visited as a guest during the Trimester, started the project *Fluctuations for the stochastic Becker-Döring process* together. This work is still in progress and it benefit from the interaction with participants of the trimester.

The visit of Gianmarco Bet gave Luisa Andreis the opportunity to start a promising and long-lasting collaboration on random graphs and coagulation processes. This is currently active and aims at understanding coagulation processes from a large deviations point of view using random graphs techniques.

Luigi Amedeo Bianchi fleshed out a paper with Dirk Blömker, a speaker at the workshop, concluded in the following months and now published [2]. The paper studies the impact of additive Gaussian white noise on the Swift Hohenberg equation with polynomial nonlinearity in an unbounded domain. The paper uses approximation via modulation equations to analyze how the influence of the noise on the dynamics close to the pitchfork bifurcation.

Luigi Amedeo Bianchi, Michele Coghi, Mario Maurelli and Margherita Zanella, together with Francesco Grotto (guest of the program and speaker at the workshop), have successfully applied for funding with the project *Stochastic fluid dynamics* (with leader L.A. Bianchi). This project is financed by Istituto Nazionale di Alta Matematica and is meant to support research visits, which we wanted to use to nurture the numerous collaborations started in Bonn.

The visit by Zdzisław Brzeźniak started his ongoing collaboration with Luigi Amedeo Bianchi and Mario Maurelli on energy conservation in stochastic Euler equations. The main aim is to find sufficient conditions on regularity of solutions to preserve energy, in presence of noise terms.

Luigi Amedeo Bianchi also worked on invariant measures for a stochastic dyadic model of turbulence. This formed part of a paper with Francesco Morandin, speaker at the workshop, available as a pre-print [3] and submitted for publication

Michele Coghi and Mario Maurelli brought their project [4] on mean field approximation for the 2D Euler vorticity equation to conclusion. The paper proves that the Euler equations driven by transport noise can be approximated by interacting point vortices driven by a regularized Biot-Savart kernel and the same common noise. The approximation happens by sending the number of particles N to infinity and the regularization in the Biot-Savart kernel to 0, as a suitable function of N.

Michele Coghi and Mario Maurelli engaged in fruitful discussions on their ongoing project on mean-field for Euler equations with Kraichnan Noise.

Mario Maurelli and Margherita Zanella started an ongoing collaboration with guest Francesco Grotto on vortex approximation for 2D Navier-Stokes in bounded domains. The main issue in this collaboration is to describe the vortex creation at the boundary, which is a peculiar fact of incompressible 2D fluids in bounded domains.

Margherita Zanella benefited from the visits of guests Zdzisław Brzeźniak and Benedetta Ferrario to work on the ongoing project on uniqueness of the invariant measure for the two dimensional stochastic Navier-Stokes equations in unbounded domains.

The visits of Zdzisław Brzeźniak and Benedetta Ferrario also made possible to start an ongoing collaboration with Margherita Zanella on existence and uniqueness of invariant measures for the two-dimensional stochastic damped Schrödinger equation in bounded domains.

Alden Waters took advantage of the time spent at HIM to work on a paper (with A. Strohmaier) [6], concerning some properties of reachable sets for the heat equation. She also made progress on another paper (in collaboration with F. Hanisch and A. Strohmaier) [5], on effect of object scattering on the Krein spectral shift formula. Both papers are available as pre-prints and have been submitted for publication.

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