

Report on the Trimester Program

Evolution of Interfaces

January - April 2019

Organizers: László Székelyhidi (Leipzig), Georg Weiss (Duisburg-Essen)

Topics

The programme focussed on the analysis of partial differential equations which involve the appearance and evolution of interfaces. The topics spanned a large spectrum from partial differential equations, geometric measure theory, convex integration and the calculus of variations. The specific themes included free boundary and obstacle problems, mean curvature flow and Brakke flow, boundary layers, mixing and instabilities in hydrodynamics.

Goals

The mathematical analysis of problems involving an evolving interface has a rich history, but it is only recently that various breakthroughs allowed one to go significantly beyond situations where comparison principles and elliptic techniques are readily available. Examples include recent progress in the theory of higher co-dimensional minimal surface theory, the analysis of singularities in free boundary problems and in the discovery of h-principle phenomena induced by fluid instabilities. These came from different areas of specialization such as on geometric measure theory, partial differential equations and convex integration, which profited from entirely different points of views. One of the main stated goals of the programme was to bring together leading experts from these different communities and, especially, to allow younger postdocs and PhD students, who have already obtained a certain amount of expertise and specialisation in one area, to learn about these neighbouring fields and profit from an exchange of ideas resulting from different points of views.

Organization

- The programme started with a winter school in the first week (January 7-11), with 4 minicourses of 4x90min each. Camillo De Lellis (IAS Princeton) gave a series of lectures on the *Regularity of area-minimizing currents*, Felix Otto (MPI Leipzig) gave a series of lectures on *Convergence of the thresholding scheme for mean curvature flow*, Yoshihiro Tonegawa (Tokyo Institute of Technology) gave a series of lectures on the *Brakke flow*, and Daniel Faraco (UAM Madrid) gave a series of lectures on *Convex integration and mixing flows*.
- Workshop on Geometric Measure Theory and Free Boundary Problems (February 11-15), consisting of 15 hour-long lectures ranging in topics such as the obstacle and thin obstacle problems to geometric Ginzburg-Landau and models for charged droplets.
- Workshop on Interfaces and Instabilities in Fluid Dynamics (March 18-22) consisting of 14 hour-long lectures ranging in topics from two-phase Newtonian and ideal fluid models to fluid instabilities and vortex sheet evolution.

Apart from these three “concentration periods”, we had a weekly seminar with external speakers as well as several weekly reading seminars. These were scheduled to take place before the 4pm tea, which was the social highlight on a daily basis.

Results

Until today, 17 preprints have originated from the program (listed on the homepage of the Hausdorff institute), including some by PhD and Master students, and further results are being written down.

Examples of the contribution of PhD and Master students are joint works of Simon Eberle with Henrik Shahgholian and Georg Weiss as well as joint works of Francisco Mengual and Florent Noisette with László Székelyhidi. The former proves a more than twenty year old conjecture on the structure of global solutions of the obstacle problem in dimension $N \geq 6$ and had not been planned before the three authors met at HIM. The work of Mengual provides a completely new approach to the problem of evolution of vortex sheet, using a combination of techniques from convex integration as well as

nonlocal nonlinear evolutionary PDE, very much in line with the philosophy of the programme.

Several junior participants continued working on topics which they first encountered during the programme. Furthermore, we have received highly positive feedback from several colleagues in the US and around the world, who during the recent covid-lockdown period have immensely profited from and based online seminars on the 4 minicourses of our winter school, which was recorded and made available online by HIM. As such, the main achievement of the programme was dissemination of ideas, especially among younger participants.

Acknowledgements

We thank the Hausdorff Institute for Mathematics for hosting our trimester program and financial support, in particular we thank the director Christoph Thiele and the friendly and very efficient staff.