REPORT ON TRIMESTER PROGRAMME AT HIM
LOGIC AND ALGORITHMS IN GROUP THEORY

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Contents

Groups and their actions constitute a formal counterpart of the intuitive notion of symmetry. Because of the importance of symmetry, group theory is one of the most developed and ramified areas of mathematics: it reaches all the way from the investigation of finite groups to continuous actions of Polish groups. Group theory is connected to most other areas of mathematics, not only via group actions but also since in many interesting cases automorphism groups preserve, sometimes complete, information about the underlying object.

1. Background

The general plan of the trimester programme at the HIM was to connect two approaches to group theory that each use tools from areas outside the immediate discipline: the *model theoretic* and the *computational* approach. These approaches had been pursued independently for a long time; despite the potential for cross-fertilisation, they had not interacted much so far.

We provide two instances of the model-theoretic approach. Model theoretic work on pseudofinite groups (infinite groups that satisfy all the first-order properties of the finite groups) has shown its relevance to the study of finite groups. Pseudofinite groups describe the asymptotic behaviour of families of finite groups. An example here is the study of approximate subgroups by Hrushovski [15] and Breuillard, Green and

Tao [4]. For further examples, see Liebeck, Macpherson and Tent [16] on classes of finite primitive permutation groups, and Macpherson and Tent [20] on word maps.

The algebraicity conjecture (also known as the Cherlin-Zilber conjecture) states that each infinite simple group of finite Morley rank is an algebraic group over an algebraically closed field. Most approaches towards proving it imitate the methods of the classification of the finite simple groups. For more information see the book [1] by Altınel, Borovik and Cherlin, and also its review by Macpherson [19].

For the computational approach, connections between group theory and computation can be traced back to Galois's work (1832) where permutation groups were used to obtain formulas that calculate solutions to certain polynomial equations. Dehn (1911) formulated the word, conjugacy and isomorphism decision problems for finitely presented groups, and gave an algorithm for solving the word problem in surface groups.

In computational group theory, a group is given in encoded form so that one can use it as an input to algorithms. It can be given by a presentation, as a set of generators that are matrices or permutations, or, in the finite case, as a black box group in the sense of [2]. For more background see [14, 24]. Interestingly, the black box groups are used in [1] to obtain proofs for special cases of the algebraicity conjecture.

2. Goals

A unifying principle of the programme was that better algorithms may help answer test questions and thus guide intuition (as has already been the case with finite groups), and conversely that a better understanding of group theoretic properties will help improve algorithms for computing in these groups.

Short presentations and short descriptions. The constructive recognition problem for black box groups has been developed by Borovik, Kantor, O'Brien, and others; see [3, 23], and [24] for background. A goal of the programme was to compare this with the problem of describing a group in a formal language via a short description (data compression). In both cases we have a recognition problem; the first is computational, the second model-theoretic.

A further goal was to compare the compression rate for finite groups with respect to two types of descriptions: finite presentations, and descriptions in first-order logic.

Guralnick, Kantor, Kassabov and Lubotzky [12] proved that all finite quasisimple groups of Lie type and rank n over a field of size q have presentations with at most nine generators and 49 relations, and bit-length $O(\log n + \log q)$, with the possible exception of the Ree groups ${}^2G_2(3^{2e+1})$. For this series of possible exceptions, the shortest known presentation has length about $\sqrt{|G|}$. A major challenge is to construct a "short" presentation for this group. Hulpke and Kassabov made some progress on this problem during the programme. On the other hand, with descriptions in first-order logic, compression down to $O(\log |G|)$ is possible [22].

Complexity of the isomorphism problem for classes of groups. Dahmani and Guirardel [6] have recently shown that Dehn's isomorphism problem is decidable for the class of hyperbolic groups. The decidability of the isomorphism problem for the larger class of automatic groups remains open.

Descriptive set theory (see [10]) can determine the relative complexity of analytic equivalence relations. Using Lubotzky [17, Prop 6.1], the isomorphism problem for groups that are finitely generated as profinite groups is Borel equivalent to the identity on $\mathbb{R}$ (smooth). The isomorphism problem for general separable profinite groups is Borel equivalent to the graph isomorphism problem by work of Kechris, Nies and Tent. For many other interesting classes of closed subgroups of S_∞, the complexity of the isomorphism problem is open. The programme addressed the problem for the class of oligomorphic groups (closed subgroups of S_∞ such that for each n there are only finitely many n-orbits).

Small cancellation theory. Tent suggested a new approach to small cancellation theory which is more recursive in nature than the original one using word length. In this approach one looks for relators not containing more than half of any other relator. Small cancellation has been a crucial technique in geometric and algorithmic group theory. The programme explored the new approach in this context.

Pseudofinite groups, and approximate subgroups. In the area of word maps, Macpherson and Tent [20] noticed that the model theoretic simplicity of the theory of certain ultraproducts of finite groups yields a new proof of some results on word maps previously obtained by Shalev [25]. One such result is that for any three non-trivial words w_1, w_2, w_3, every sufficiently large finite simple group G satisfies $G = w_1(G)w_2(G)w_3(G)$.

In a different direction, a conjecture of Babai states that the diameter of the Cayley graph of a finite simple group G with respect to any generating set is $O(\log^c |G|)$ for some constant c. Recent progress was made by Helfgott (see his survey [13]) for the case of $SL_2(q)$, and hence $PSL_2(q)$, where q is a prime.

The programme explored the computational implication of this work.

Automorphism groups of free groups. The structure and properties of the automorphism group of a nonabelian free group is a topic with many open problems, some of them susceptible to computer investigation. For example, it is unknown whether $\mathrm{Aut}(F_n)$ has the Kazhdan Property (T) for $n \geq 4$. An answer could provide insight into the behavior of the *Product Replacement Algorithm* (see [24, p. 678]) for choosing random elements of large finite groups. It performs better than is indicated by theoretical analysis. For if the automorphism group has Property (T), then the graph defined by the algorithm is an expander and the associated random walk on the group is linear in $\log |G|$; for further discussion, see [18].

Work of Grunewald and Lubotzky [11] shows that the representation theory of $\mathrm{Aut}(F_n)$ is rich. They offer an algorithmic approach to constructing generators for certain subgroups of "large" finite index of this group, and associated representations exhibiting interesting commensurable properties. They pose a number of challenging open problems about these representations. Their approach allows computational investigation of whether $\mathrm{Aut}(F_4)$ satisfies Property (T) by searching for a subgroup which has infinite abelianization. Essentially identical techniques can be applied to study whether the mapping class group of closed surfaces of genus at least 3 satisfies Property (T).

The coclass of a group of order p^n and nilpotency class c is defined to be $n-c$. Pro-p groups have played a critical role in the classification of finite p-groups of a fixed coclass. An interesting development was the affirmative answer by du Sautoy [8] of "Conjecture P" in [21] using model theory. This shows that the classification up to isomorphism of 2-groups of a fixed coclass is a "finite" (but essentially impossible) task; the equivalent, if much more elaborate, conjecture for odd primes and its proof for $p = 3$, involving extensive study of certain pro-3 groups, appeared in [9].

The programme also addressed potential connections between classification of p-groups and the work of Nies and Tent [22] which show that, when considering the length of descriptions in first-order logic, among the finite groups the p-groups are among the hardest to describe.

A final set of goals concerned algorithms for decision problems, and their performance:

Linear groups. Following recent advances in the development of theoretical and practical algorithms for computing with large finite linear groups [23], the focus is moving to finitely generated infinite linear groups. The *Tits Alternative* provides an initial dichotomy: either there is a solvable subgroup of finite index, or there is a nonabelian free subgroup. We can now sometimes algorithmically distinguish between these cases [7]. The situation in which the group has a nonabelian free subgroup is the more challenging. A major problem here is, under suitable additional assumptions, to decide algorithmically whether a finitely generated subgroup is finitely presentable [5]. Deciding constructively whether a finitely presented group is linear is equally important.

Verifying hyperbolicity and other properties. An important open question in geometric group theory is whether all hyperbolic groups are residually finite. Following the breakthroughs of Agol and Wise, we now know that every 3-manifold group is residually finite and that every hyperbolic 3-manifold group G has a subgroup of finite index that is special. The problem of determining the least index of such a subgroup (or the nature of the minimal finite quotients of G) is open and ripe for exploration using computational methods.

One approach to the residual finiteness problem was to analyze random hyperbolic groups. Holt, Neunhöffer, Parker and Roney-Dougal are developing practical algorithms and related software which attempts to verify the hyperbolicity of groups defined by random presentations with very large numbers of generators. In addition to its possible use in finding random hyperbolic groups, this software should lead to improved insight into the theoretical and practical difficulty of solving the word and conjugacy problems in such groups.

Topological methods in algorithmic group theory. There are some heuristic methods for computing in groups arising in a topological context, such as fundamental groups of 3-manifolds, that run very fast in practice without any apparent theoretical justification. We investigated these methods, seeking to both understand the parameters which underpin their performance and to develop generalizations of these techniques.

References

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3. Organization

Since the proposal covered a wide range of topics - unified by a common theme - we arranged mini-courses as introductions to the contributing areas to facilitate the interaction. This was important not only for the junior participants, but also for senior participants from different fields. Each course ran over one week and consisted of about 4-6 hours of lectures per week. The courses were grouped into thematic lecture series of which there were 9 during Sept-October, with 2 courses each. Many courses also included informal discussion sessions. Lecture series speakers included Henry Wilton, Martin Bridson, Harald Helfgott, Colva Rooney-Dougal, Gaven Martin and Katrin Tent.

The main workshop at the end of October covered the full breadth of the programme. There was a weekly seminar, and also a popular series of 2-3 about 30 minute talks on Thursday by the younger participants. This programme was carefully supported by the HIM staff, in particular Stefan Hartmann. To collect and share material there was a common dropbox; to help organize there was also a google calendar. Both were maintained by the junior participants of the programme.

A one-week school at the beginning of the programme was especially geared at students, with introductory courses:

- Undecidability in groups by Maurice Chiodo (Cambridge),
- Representation theory for groups of Lie type by Gerhard Hiss (Aachen),
- Algorithms for finitely-presented groups by Derek Holt (Warwick),
- Pseudofinite groups by Dugald Macpherson (Leeds),
- Stability and invariant random subgroups by Andreas Thom (Dresden).

4. Some experiences and results described by participants

Bays. I stayed for the first two weeks of the programme. I talked with a number of people about a number of things, and attended many of the lectures in the summer school of the second week, but my primary activity was discussion with Itay Kaplan. We further developed a research project joint with Pierre Simon on distal phenomena in arbitrary NIP theories, and in particular on the behaviour of a related isolation notion for types known as compressibility. We have some results on this by now, and are in the process of preparing a paper. The discussions in Bonn certainly pushed the project forward and helped to orient us.

I also had some entertaining discussions with Greg Cherlin on a number of topics, and particularly on the question of the tameness or otherwise of the elementary theory of an ultraproduct of finite groups of Lie type of increasing rank but fixed characteristic. No concrete output is expected from this, but it certainly contributed to my understanding. I had other useful interactions with various people; certainly I remember my visit as a pleasurable and stimulating experience.

Borovik. The most important outcome was further consolidation of links between the model theory and black box algebra, helped by conversation with Gregory Cherlin who also participated in the programme. See reference below.

Bradford. Participating in the program enabled me to push forward his ongoing collaboration with Andreas Thom concerning the structure of laws satisfied by finite groups, leading to the completion of our paper 'Short laws for finite groups of Lie type' (arXiv:1811.05401) which appeared on arXiv during the program.

I made substantial progress on a project concerning local embeddings of groups into finite groups, particularly helped by discussions with Remi Coulon, Alexander Hulpke and Henry Wilton, which will culminate in a paper "Quantifying local embeddings into finite groups" (to appear).

Brooksbank and Leemans. We worked on rank reduction techniques for string C-group representations leading to paper. We continue to work to find rank augmentation techniques.

Bridson and Reid. The main research activities involved continuation of a long term ongoing project on to what extent finitely-generated residually finite groups are determined by their profinite completions.

They completed the paper: M.R. Bridson, D. B. McReynolds, A. W. Reid, R. Spitler. "Absolute profinite rigidity and hyperbolic geometry" arXiv:1811.04394

This paper is the culmination of many years of work on one of the basic questions in the field of profinite rigidity: are there finitely presented, residually finite groups that are full-sized (ie contain a non-abelian free group) but can be distinguished from all other finitely generated, residually finite groups by means of their finite quotients.

They also worked on a related paper close to completion. M.R. Bridson, D. B. McReynolds, A. W. Reid, R. Spitler. “On the profinite rigidity of triangle groups”.

Carnevale. Paula Lins and I discussed combinatorial properties of Lins's formulae for the conjugacy class zeta functions of some group schemes. The idea is to use a theory of Hadamard products that I am currently developing to compute the conjugacy class zeta function of arbitrary direct products of these groups.

Cherlin. The more concrete products of the term involved joint work with Wiscons on relational complexity. None of this is in published form yet.

The main products are:

(a) A set of GAP routines which give useful information about relational complexity in permutation groups of moderate degree (typically a few hundred), and some resulting data.

(b) Detailed notes, circulated very narrowly - to Gill and his collaborators - on the main problems and conjectures in the area, together with various concrete results that clarify the problems and conjectures.

(c) Research notes with Wiscons on some significant points discovered during the period of the Trimester, also shared with Gill.

Lingering in the background is a long article with Wiscons written just before the Trimester which will probably be our next publication on the subject. I suspect that any HIM-related work will have to wait for publication till that is sorted out.

The Trimester was beneficial to our project in a variety of ways, most concretely with respect to (a) - both development and applications - though some theoretical points were cleared up as well.

Ciobanu. I had significant interactions with Ben Martin (Aberdeen), Meng-Che Turbo Ho (Purdue), Remi Coulon (Rennes), and Olga Kharlampovich (CUNY).

The discussion with Ben Martin was related to zeta functions of groups, and whether these would be suitable to study in the context of types of growth other than subgroup or representation growth. We concluded that the standard techniques from subgroup growth zeta functions for nilpotent groups do not apply to conjugacy growth zeta functions, for instance. However, this discussion raised the potential of different kinds of zeta functions, such as the one introduced by Gromov for hyperbolic groups.

The discussion with Turbo Ho included my PhD student Alex Evetts. Also Alex, Turbo and I completed a paper together on the conjugacy growth series of soluble Baumslag-Solitar groups. This was perhaps the most exciting development of my stay, as we could combine different, complementary, sets of techniques to work out fairly quickly how to handle conjugacy series for these groups.

Conversano. I greatly benefited from my participation in the HIM Trimester. The Summer school, Lecture Series and the International Workshop all had excellent speakers, allowing me to learn directions of the research in group theory that I was not familiar with (like algorithmic and geometric aspects), and improve my understanding of the logic components I am mostly interested in.

Besides having many useful and very interesting conversations with several participants to the program, during my time at HIM I started ongoing collaborations with Gaven Martin on commutators in simple real Lie groups, and with Andre Nies on the expressivity of first-order logic for Lie groups. With Nies we have preliminary results on abelian and compact groups, and we expect the research to continue with the focus towards simple groups.

The excellent facilities, including a very comfortable office, allowed me to work also on personal projects related to the topics of the program, now completed in the articles listed in the next section.

Deloro. The stay resulted in the completion of arxiv preprint 1901.04453 which will appear in BLMS. It also included substantial advances towards another preprint, which will be posted in a few weeks, on revisiting the representation theory of Sym(n) and Alt(n) in a most elementary way, without linear algebra. In general, meeting with T. Altinel, A. Berkman, and G. Cherlin also helped me understand other topics.

Detinko. I worked on new methods, algorithms, and software for practical computing with Zariski dense linear groups. We explored applications of our software to solution of open mathematical problems by com puter experimentation. This includes experiments with hypergeometric groups which appeared at the interface of differential Galois theory and theoretical physics.

Eick. I carried out work with Eamonn O'Brien, Gabriele Nebe and Stefano Marseglia on conjugacy problems in GL(n,Z). Ongoing work with Subhrayoti Saha on aspects of infinite pro-p-groups and GAP implementations to investigate these.

Gardam. During the programme I thought about a lot of different things, due to the constant flow of visitors and talks. There were 4 major projects:
1. trying to bound the complexity of the isomorphism problem for rigid hyperbolic groups, together with Michal Ferov during his visit;
2. working on various aspects of one-relator groups, together with Alan Logan during his visit;
3. a project on torsion in homology of finite covers of hyperbolic 3-manifolds;
4. an existing collaboration on Cannon–Thurston maps for CAT(0) groups.

The arxiv preprint 1810.13285 is the result of project 4, projects 1, 2 and 3 mentioned above are ongoing / incomplete.

Kaplan. During my stay at the HIM, I mostly collaborated with Martin Bays who visited for the first two weeks of my stay. We started this cooperation on the problem we worked on (which was related to NIP theories in abstract model theory) back in January of 2018, but the approach we tried then did not seem to converge. Here we tried looking at the problem again, and we seemed to have gained some new information on the problem that might lead to an approach, or hopefully to some new insight. After discussing this with Pierre Simon, we discovered that some of the results we proved were already known by previous works, but the proofs are a bit different, so I am still cautiously optimistic. I also had very nice conversations with Greg Cherlin and Andre Nies about various subjects regarding homogeneous structures and with Annalisa Conversano about groups in o-minimal structures.I also attended the summer school which was very nice.I enjoyed my stay at the HIM very much, it is a very peaceful, quiet place and it offers supreme conditions for doing research. The staff here was very helpful, and every little problem I had, whether it was technical support or help with the apartment, I could rely on them for help.

Kharlampovich. During my short visit I collaborated with Remi Coulon on questions related to the growth of balls in free groups in the varieties with identity $x^n = 1$ for large odd n (free Burnside groups) and on Random Burnside groups.

Ben Martin. I worked on a project with Michele Zordan on the representation theory of a compact p-adic analytic group. We proved some results on the geometric properties of Lie shadows. We expect to finish writing up this work in the next few months.

Nebe. I had informal discussions with several participants, in particular Alla Detinko (on the congruence subgroup problem for S-arithmetic groups), Alexander Hulpke, Bettina Eick and Eamonn O'Brien (in particular on conjugacy of integral matrices).

Nies and Tent. We continued work on two opposite classes of topological groups: oligomorphic groups and profinite groups. For the first class, with Schlicht we obtain an upper bound on the complexity of topological isomorphism. For the second class, we proved results in the direction unique description via a first-order sentence. Two papers are on arxiv, see references below. Nies also worked with Kaplan and with Conversano on question connecting model theory and group theory.

Rees. I collaborated with Barbara Baumeister and Georges Néaimeé. We studied groups defined from posets of divisors of quasi-Coxeter elements that are associated with Artin groups of spherical type. With Henry Bradford we considered some combination theorems for sofic groups.

Rossmann. Carnevale and I began a collaboration on rank distributions in spaces of matrices with combinatorially-defined linear constraints. Applications include the computation of conjugacy class zeta functions associated with free class-3 nilpotent groups, generalising work of O'Brien and Voll. An article based on this research is currently being written up.

Voll and I collaborated on the study of a class of rational generating functions associated with graphs. This work will form part of a sequel to their article "Groups, graphs, and hypergraphs: average sizes of kernels of generic matrices with support constraints" (arXiv:1908.09589).

During several mathematical discussions, Joshua Maglione, James Wilson and I explored connections between tensor singularities and the geometric essence of orbit-counting and conjugacy class zeta functions of groups.

Tent With Rips and Atkarskaya I worked on topics related to the Burnside problem. Upcoming work with Segal started during the program deals with algebraic groups over certain rings, and the question whether bi-interpretability holds between the ring and the group.

Thomas. During my stay at the HIM, I presented a lecture series on Invariant Random Subgroups (IRSs) of countable groups, which included an account of Vershik's classification of the ergodic IRSs of the group Fin(N) of finite permutations of the natural numbers N. While at the HIM, I studied the relationship between the ergodic IRSs of the group Fin(N) and the indecomposable characters of Fin(N). This led to an interpretation of each indecomposable character as an asymptotic limit of a naturally associated sequence of characters induced from linear characters of Young subgroups of finite symmetric groups. This research resulted in a paper entitled "Characters and invariant random subgroups of the finitary symmetric group", which has been submitted for publication.

Voll. I engaged in discussions with numerous colleagues (including O'Brien, Eick, Nebe, Rees) on a range of topics, including finite p-groups, polycyclic groups, invariant theory. With Eamonn O'Brien, in particular, I advanced an ongoing research project on automorphism groups of finite p-groups.

Zordan. I wrote up a joint result with Stasinski applying model theory to representations zeta functions (the pre-print is available on his website and it will soon be on ArXiV after they have finished adding a new result).

To end, here is some feedback from a participant which might summarize the experience of many of the contributors to the programme: "I just read over my notes from HIM summarizing what I did every day. I was reminded of interesting discussions I had with dozens of people that were very helpful to me, even if they didn't deliver something that can now be pinned down on a particular preprint or proof. Most of the projects I focussed on during the programme are incomplete, but I think the greatest benefit of such a programme is what one learns through all these small interactions, even if it is so hard to quantify. Whether it was learning about important examples of tree lattices from Alex Lubotzky, asking Laura Ciobanu about her work on solving equations, discussing curious GAP behaviour with Alex Hulpke (who then fixed the responsible bug in short order), or picking Alan Reid's brain about 3-manifolds, all of these encounters have left their mark on me."

5. Preprints and papers

(1) Daniela Amato, Gregory Cherlin and Dugald Macpherson. Metrically Homogeneous Graphs in Diameter 3. Accepted to appear J. Mathematical Logic.
(2) Alexandre Borovik and Şükrü Yalçınkaya. Natural representations of black box groups encrypting $SL_2(\mathbb{F}_q)$. arxiv:2001.10292v2.

(3) M. R. Bridson, D. B. McReynolds, A. W. Reid, R. Spitler. Absolute profinite rigidity and hyperbolic geometry. arXiv:1811.04394
(4) P. A. Brooksbank and D. Leemans. Rank reduction of string C-group representations. Proc. Amer. Math. Soc. **147** (2019), 5421–5426.
(5) Peter Brookbank, Joshua Maglione, and James Wilson. Rosenberg-Zelinsky sequences for tensors. J. Algebra 545 (2019) 43-65.
(6) Brooksbank, Grochow, Qiao and Wilson. Weisfeller-Lehman algorithms for Group Isomorphism. (arXiv:1905.02518)
(7) Henry Bradford and Andreas Thom. Short Laws for Finite Groups of Lie Type. arXiv:1811.05401 [math.GR]
(8) Laura Ciobanu, Alex Evetts, and Turbo Ho. The conjugacy growth of the soluble Baumslag-Solitar groups. https://arxiv.org/abs/1908.05321.
(9) William Cocke and Turbo Ho. Word Maps in Finite Simple Groups. To appear Archiv der Mathematik.
(10) Remi Coulon and Alan D. Logan and Turbo Ho. The universal theory of random groups. Submitted.
(11) A. Conversano. Nilpotent groups, o-minimal Euler characteristic, and linear algebraic groups. arXiv:1904.09738.
(12) A. Conversano. Definable groups in o-minimal structures: various properties and a diagram. Accepted to appear in Research Trends in Contemporary Logic, College Publications London.
(13) Adrien Deloro and Joshua Wiscons. The geometry of involutions in ranked groups with a TI-subgroup. To appear Bull. London Math. Soc.
(14) A. Detinko, D. Flannery and A. Hulpke. Experimenting with hypergeometric symplectic groups. arxiv.org/abs/1905.02190.
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(16) Heiko Dietrich and James Wilson Isomorphism testing of groups of cube-free order. J. Algebra 545 (2019), 174–197.
(17) Heiko Dietrich and James Wilson Polynomial-time isomorphism testing of groups of most finite orders. arXiv:1806.08872.
(18) Bettina Eick, Tommy Hofmann and E.A. O'Brien. The conjugacy problem in $\mathrm{GL}(n, \mathbb{Z})$. J. London Math. Soc. 100 (2019), 731–756.
(19) Robert Gilman. Algorithmic search in group theory. J. Algebra 545 (2020), 237 – 244.

(20) S. P. Glasby, C. E. Praeger, and Colva Roney-Dougal. Involution centralisers in finite unitary groups of odd characteristic. J. Algebra 545, (2020).
(21) Derek F. Holt, C.R. Leedham-Green, and E.A. O'Brien. Constructing composition factors for a linear group in polynomial time. J. Algebra 2020.
(22) Ian Holm Kessler, Henry Kvinge and James Wilson. A Frobenius-Schreier-Sims Algorithm to tensor decompose algebras. arXiv:1812.03346.
(23) Kassabov, Tyburski, Wilson: The number of isomorphism classes of subgroups of finite simple groups of Lie type is maximum possible. In preparation.
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