Report on the HIM special program

Applied and Computational Algebraic Topology

April 24 - May 6 and September 4-29, 2017

ORGANIZERS:

- Herbert Edelsbrunner (IST Austria)
- Kathryn Hess (EPFL, Switzerland)
- Michael Farber (Queen Mary, University of London, UK)
- Dmitry Feichtner-Kozlov (Universität Bremen, Germany)
- Martin Raussen (Aalborg Universitet, Denmark)

1. TOPICS AND GOALS

Scientific and engineering disciplines are in increasing need of effective methods for distilling and analyzing structural information from vast quantities of complex data. In recent years, ideas and methods from algebraic topology, combined with the development of fast algorithms and user-friendly software, have proven valuable in this endeavor. The program aimed at stimulating and enhancing collaboration between experts in several areas of applied and computational topology, together with scientists actively working on problems, the solution of which might require topological insights and machinery. It focussed on the following three areas of investigation.

- Topological and statistical analysis of shapes, images, and multi-dimensional data sets
- Stochastic topology
- Topological contributions to the theory of concurrent computation and computer networks

2. ORGANIZATION OF THE PROGRAM

The HIM program started with a week-long spring school (April 24 – 28, 2017) consisting of 14 lectures (90 minutes each, including a break and time for questions) aimed at graduate students and colleagues from different fields of expertise. These lectures introduced and developed core topics related to the three themes mentioned above. The schedule and the titles of the talks, as well as the slides and recordings (for most of the talks), can be found on the program webpage. The spring school had 58 participants, including the lecturers.

The subsequent week (May 2 – 6, 2017) was devoted to a conference bringing together both experts and several of the participants in the spring school. Twenty speakers gave advanced lectures (duration 50 minutes each) on their fields of interest; the breaks gave rise to many stimulating discussions. A poster session allowed young participants to present their research efforts, and a software session provided insight into up-to-date implementations. Abstracts of all talks and recordings of most of them are available on this webpage. The conference had 58 participants.

The last part of the program consisted in a four-week period (September 4 – 29) devoted to collaboration between experts. It was purposefully designed with a sparse program of talks, to give the participants time to work together, most often in small groups. Nevertheless, the 52 participants, who stayed for anywhere from a couple of days to the entire program, and a few guests had the possibility of attending 15 seminar lectures. While the first week had no specific topic (it featured, among others, an interesting glimpse into the rising field of neuro-topology, bringing topology and neuroscience into interaction), the three remaining weeks focused on

- distributed computing and concurrency,
- topological data analysis, and
- stochastic topology.

3. Results and Publications

The special program gathered a considerable number of researchers from all over the world, working in a variety of areas of applied and computational topology. The talks and, even more so, subsequent discussions, gave rise to numerous new collaborations and provided new impetus to existing collaborations. The program led also to contacts between young students and their future Ph.D.-advisors. So far 32 research papers have appeared (some already published in journals, many of them on the arXiv, and a few not quite in final form) that are clearly related to the special program. As a side effect, the arrangement of the successful 2018 GETCO (Geometric and Topological methods in COmputer Science) conference in Oaxaca, Mexico, was planned during the special program.

Below we list some examples of collaborations that substantially benefitted from the special program, chosen from different subareas.

- To further the study of questions in stochastic topology, we invited Daniel Hug to deliver lectures on topics in the related field of stochastic geometry. This was helpful in leading to the solution of old geometric questions with new topological methods. One example is the expected number of p-simplices in the Delaunay mosaic of a Poisson point process in n-dimensional Euclidean space. Edelsbrunner and Nikitenko computed expressions for the expected number of intervals of the radius function on the Delaunay mosaic, which is a generalized discrete Morse function. Using these expressions, they computed the expected number of simplices for all $p \leq n \leq 4$. Prior to this work, the expected numbers were known for all $p \leq n \leq 3$ from classic work of Miles in 1970. Extending these results, Edelsbrunner and Nikitenko computed similar expectations for weighted Delaunay mosaics and for order-k Delaunay mosaics.
- The theme of stochastic topology was continued in talks dedicated to random simplicial complexes. Such complexes are high-dimensional generalizations of random graphs, which are a very popular tool in applied mathematics and in

other sciences. Random simplicial complexes of high dimension appear in various applications. For example in the theory of networks, simplicial complexes of dimension ≥ 2 describe interactions among 3 or more objects.

The talk by Farber described results about the Betti numbers and fundamental groups of a certain class of large random simplicial complexes. The behavior of the Betti numbers may seem surprising: the largest Betti number occurs in a specific dimension (called *the critical dimension*). The Betti numbers vanish below the critical dimension and are very small above the critical dimension. The properties of the fundamental group are also unexpected: the fundamental group is either free or has Kazhdan's property (T).

The talk by Meshulam described different constructions of random simplical complexes (using Latin squares) which possess remarkable properties, such as being high dimensional expanders.

- An emerging area of applied topology deals with combinatorial methods for calculation of higher Cheeger constants. A collaboration between Roy Meshulam and Dmitry Kozlov, in part initiated and substantially furthered by the special program, aims at developing new tools in this area. Multiple new results were produced, starting from new estimates for the Cheeger constant of a simplex, and ranging to developing a new "detecting cycle" technique for producing such estimates in general.
- Following up the investigation of topological complexity introduced by Farber a decade ago, Goubault and Farber defined and investigated directed topological complexity of directed spaces. Ziemiański and Raussen, in response to an eye-opening example from Dubut's thesis, each developed new definitions for components of directed spaces and investigated their properties. Goubault, Farber, Sagnier, and Raussen discussed and suggested definitions for directed topological homotopy equivalence, a surprisingly subtle topic concerning a good notion of symmetry (up to homotopy) for directed spaces.
- Directed flag complexes and other complexes arising from directed networks are essential tools in the study of neural networks, such as the digital reconstructions of brain microcircuitry created by the Blue Brain Project. The *Ripser* package for efficient calculations of persistent homology was presented by Bauer during the HIM program. Inspired by *Ripser*, a new computing package *Flagser*, optimized for very large datasets, and designed to construct the directed flag complex of a directed graph and to compute persistent homology for suitably defined filtrations on the graph and on the resulting complex, was created subsequently by Lütgehetmann, Govc, Smith, and Levi.