Report on the Hausdorff Trimester Program

Von Neumann Algebras

May - August 2016

Organizers: Dietmar Bisch, Vaughan Jones, Sorin Popa,
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Topics

The theory of von Neumann algebras has seen many spectacular developments in the last ten years. Three main directions have emerged: Jones theory of subfactors, Voiculescus free probability theory and Popas deformation-rigidity theory. Each of these directions has produced amazing results, surprising applications and deep interactions with other areas of mathematics and theoretical physics.

Goals

This program aimed at bringing together researchers in these sub-areas of the theory of von Neumann algebras, exploring connections among them and investigating new directions.

Organization

The main organizational activities were

- an introductory school, combined with the annual Spring Institute on Noncommutative Geometry and Operator Algebras. Funding for this event was provided by NSF $(42k), Vanderbilt$(25k) and HIM (?). Around 120 participants from all over the world came to this event, the majority students and postdoctoral researchers. Thanks to the staff support provided by HIM, the school/conference ran very smoothly and the videos of the talks are readily available online.

- a one week workshop "Von Neumann Algebras" at HIM

- a bi-weekly/weekly research seminar (Trimester Seminar) with several invited speakers who came to discuss their work and collaborate with researchers present at HIM

- a weekly seminar (Junior Seminar) organized by students and post-docs, which was combination of a research seminar (young researchers presented their first results) and a learning seminar (e.g., Brent Nelson led a learning module on Tomita-Takesaki theory).

Research Highlights

- Bisch worked with his collaborators Ghosh and Rakshit at HIM on determining Ocneanu's tube algebra for the so-called Bisch-Haagerup subfactors with 3-cocycle. This class of subfactors provides an exciting source of examples of infinite depth subfactors with interesting representation categories of bimodules. The work might very well be key to a future study of infinite depth subfactors via their associated rigid C*-tensor categories with infinitely many simple objects. The results appeared in a joint paper of the above-mentioned authors and Das, see arXiv:1701.00097.

- Jones explored ideas of how to construct a conformal field theory from a subfactor or a planar algebra and made significant progress. There has been interest in the semicontinuous limit of a quantum spin chain. This limit is obtained by reversing the process of block spin renormalisation a la Kadanoff/Wilson. Thus, for instance, the spins in a chain of length 4 are all doubled, by the adjoint of a spin blocking operator, to obtain a chain of length 8, and so on. If the spin doubling operator is an isometry one obtains a pre-Hilbert space as the inductive limit. This procedure was proposed as a possible way of obtaining a chiral conformal field theory from a subfactor. The spin doubling operator can be chosen from the planar algebra of a subfactor if one allows anyonic spin chains.The Thompson groups F and T were supposed to approximate the diffeomorphism groups of the interval and circle respectively. Although Jones has shown that this process will not give a continuum theory, he did obtain unitary representations of the Thompson groups that are of considerable interest, and it is still possible that there are applications to physics. Visits to HIM by Juerg Froehlich and Tobias

Osborne were very productive in this regard. The proof of Joness no-go result asserting that the rotations in T act highly discontinuously uses the transfer matrices and a certain rational map on the spectral parameter space defined by the spin blocking operator. The dynamical systems on spectral parameter space appear to be rather interesting, exhibiting rich Julia and Fatou sets. In a neighbourhood of each repelling point of the Julia set it is possible to define transfer matrices on the semicontinuous limit depending on a parameter. The definition is in the sense of quadratic forms with form domain being the inductive limit of the anyonic spin chain Hilbert spaces. Two articles have appeared on the arXiv outlining these results:

- Vaughan F. R. Jones, Scale invariant transfer matrices and Hamiltionians (arXiv:1706.00515)
- Vaughan F. R. Jones, A no-go theorem for the continuum limit of a periodic quantum spin chain (arXiv:1607.08769)

- Popa and Shlyakhtenko revised and finalized their paper "L^2-Betti numbers for quasi-regular inclusions" with Stefaan Vaes (arXiv:1511.07329) during the trimester at HIM. The paper will appear in the International Mathematical Research Notices. This is a significant paper in which the authors define L^2-Betti numbers for standard invariants of subfactors,generalizing those for groups. This is done via the symmetric enveloping von Neumann algebra associated to any subfactor. In fact, Popa, Shlyakhtenko and Vaes develop a (co-) homology theory for quasi-regular inclusions of von Neumann algebras. This allows them then to define a Hochschild-type (co-)homology theory and L^2-Betti numbers for rigid C*-tensor categories. They compute these Betti numbers for Temperley-Lieb-Jones categories and the Fuss-Catalan categories of Bisch and Jones.

- Sasyk and Tornquist show that the free Araki-Woods factors of Shlyakhtenko are not classifiable by countable structures (arXiv:1708.07496). This is the first time that it could be shown that this important and much studied class of factors is extremely complex. The proof uses Hjorth's theory of turbulence. Sasyk gave a mini-course on this result at the annual spring institute on noncommutative geometry and operator algebras at Vanderbilt in May 2017.

- Young researchers felt especially welcome at our HIM program and several preprints resulted as a consequence (e.g. the papers by S. Parekh, C. Wen, P. Bikram, K. Mukherjee, M. Bischoff, I. Charlesworth, C. Jones, B. Nelson to name just a few). For instance, Sandeepan Parekh, currently a PhD student at Vanderbilt, produced a preprint (his first paper!) "Maximal amenability of the generator subalgebra in q-Gaussian von Neumann algebras" during his stay at HIM (to appear in Journal of Operator Theory). Here are a couple of paragraphs that Parekh said about the Von Neumann algebras trimester: "I have a fond memory of the summer of 2016, which I spent working at the Hausdorff Institute of Mathematics as a participant of the Trimester Programme on von Neumann algebras. HIM had brought together a plethora of mathematicians and by being in proximity with this unique mix of people I found myself working on a project with two of my now-collaborators Chenxu Wen and Koichi Shimada. Sitting at my desk, with all sorts of office supplies within hands reach, was a pleasant experience where I could forget everything else and focus on my research. When I needed a break, I would take a walk around the very pretty park which was just outside the building. Our project involved some very tedious computations and I was especially grateful for this excellent environment provided at Bonn, Germany. Every evening there would be cake and coffee provided which brought people out from their offices. It was in one of these cake session discussions, while working away at the blackboard in the garden that me and my collaborators thrashed out the final details of our proof. All in all, it was a lovely experience and I was very happy to have had the opportunity to visit the Hausdorff Institute of Mathematics at Bonn."