

Report on the Hausdorff Trimester Program

Non-commutative Geometry and its Applications

September 1 - December 19, 2014

Organizers:

Alan Carey, Australian National University,
Victor Gayral, University of Reims,
Matthias Lesch, University of Bonn,
Walter van Suijlekom, Radboud University, Nijmegen,
Raimar Wulkenaar, University of Münster.

Scientific Advisory Committee:

Nigel Higson (Penn State), Bruno Iochum (Provence),
Henri Moscovici (Ohio State)

Topics

Noncommutative geometry (abbreviated NCG in the sequel) is the exploration of geometric concepts through operator-algebraic methods, as initiated and outlined by Alain Connes. The program had its focus on the key applications of noncommutative geometry as the organizers expected them within the near future. Additionally, the program welcomed researchers doing basic research in NCG.

The program was devoted to the following themes.

- Mathematical questions in the traditional parts of NCG such as index theory, non-commutative spaces, spectral geometry, deformations, foliations.
- Interactions with other parts of mathematics, particularly number theory/arithmetical geometry.
- Applications of NCG to physics including the standard model of elementary particle physics, condensed matter theory, Lorentzian space-time, and the spectral action principle.

Goals

The range of topics in the initial proposal was deliberately chosen to be broad with the eventual program focussing on the interests of those who accepted the invitation to participate. It was intended to have interactions between those working on the “core” topics with researchers in other parts of mathematics and also in physics who might be using non-commutative methods. There was an emphasis on the instructional program which provided a focus of activity for junior researchers for September and October.

Organization

The activities included a summer school for research students in the second week (in September) and four topical workshops spread through the program. There was an instructional series of lectures organised for October and special topics lectures in November and December. A junior seminar, which was organized by younger scientists, ran intermittently through the trimester and there were numerous seminars. A special topics seminar offered lecture series delivered by senior scientists.

A particular highlight was the Plücker lecture on “Some Aspects of Non-commutative Geometry” delivered by Nigel Higson. The Plücker lecture series is one of the prestigious named lecture series of the Bonn mathematics departments. It was announced as a joint event of the HIM program and the official mathematics departments activities.

The last workshop was deliberately oriented towards emerging topics in non-commutative geometry or potential topics of application. Additionally, there was a special seminar by Alain Connes.

The first workshop was aimed at exploring questions where non-commutative geometry ideas had been applied in other parts of mathematics. The second workshop considered applications in some parts of physics.

The third workshop, organised by Alain Connes, Katia Consani and Sergey Neshveyev, considered the interaction of non-commutative geometry methods with arithmetic geometry and number theory.

For the final workshop speakers were asked to focus not on recent new results but to provide lectures pointing to possible future applications or theoretical developments.

Results

Many collaborations were either continued or commenced during the trimester. Some of these were planned deliberately (by inviting individuals who were already working together) while others arose spontaneously as individuals discovered common interests with other researchers. The concentration of researchers at different times during the trimester in the three areas of non-commutative index theory, applications to quantum physics and finally number theory produced a series of publications (listed at the end of the report).

Some examples of the collaborations follow.

A group working on KK theory and the unbounded Kasparov product formed including Arici, Cacic, Kaad, Lesch, Mesland and Rennie. There were several publications attributable to this collaboration. A group working on index theory also formed and loosely involved Baum, Carey, van Erp, Kaad and Grosse. NCG and representation theory was studied by Higson and Yuncken whiel the latter also worked with van Erp on index theory for sub-elliptic operators.

A collabration on topological insulalators was started (Bourne, Carey, Kellendonk, Rennie) and has produced two publications to date with further promised.

Strohmaier worked with Werner Müller from Bonn and started a collaboration with Pennig an example of the many spontaneous interactions that occurred during the semester.

A topic where progress was made during the semester was NCG on Lorentzian spacetimes where there was a collaboration between van den Dungen, Eckstein and Stephan aiming for a Lorentzian version of the spectral action in noncommutative geometry.

Junior researchers such as Venselaar made great use of their time establishing new collaborations as well as meeting up with previous collaborators such as Cornelissen, Mesland and Sitarz. The concentration of financial resources on them certainly paid dividends in terms of enabling them to find collaborators both amongst the senior participants as well as their peers.

Publications

At the time of writing this report there seem to be quite a few publications that were generated by the Trimester. We list them below grouped according

the the main themes. Numbers refer to the arXiv.

Physics and NCG

- Bourne, Chris; Carey, Alan L.; Rennie, Adam, The bulk-edge correspondence for the quantum Hall effect in Kasparov theory 1411.7527
 - Boeijink, Jord; van den Dungen, Koen, On globally non-trivial almost-commutative manifolds 1405.5368
 - Lechtenfeld, Olaf; Popov, Alexander D.; Szabo, Richard J., Sasakian quiver gauge theories and instantons on Calabi-Yau cones 1412.4409
 - Franco, Nicolas; Eckstein, Michal, Causality in noncommutative two-sheeted space-times 1502.04683
 - Bourne, Chris; Carey, Alan L.; Rennie, Adam, A noncommutative framework for topological insulators 1509.07210
 - Dabrowski, Ludwik; Landi, Giovanni; Luef, Franz, Sigma-model solitons on noncommutative spaces 1501.02331
 - Kurkov, Maxim A.; Lizzi, Fedele; Sakellariadou, Mairi; Watcharangkool, Apimook, Spectral action with zeta function regularization 1412.4669 appeared in Phys. Rev.
 - Devastato, Agostino; Lizzi, Fedele; Flores, Carlos Valcarcel; Vassilevich, Dmitri, Unification of coupling constants, dimension 6 operators and the spectral action 1410.6624 appeared in Int. J. Mod. Phys. A
 - Fathizadeh, Farzad; Ghorbanpour, Asghar; Khalkhali, Masoud, Rationality of spectral action for Robertson-Walker metrics 1407.5972
 - Samary, Dine Ousmane; Perez-Sanchez, Carlos I.; Vignes-Tourneret, Fabien; Wulkenhaar, Raimar, Correlation functions of a just renormalizable tensorial group field theory: the melonic approximation 1411.7213

Index theory, KK-theory, spectral triples ...

- van Erp, Erik; Julg, Pierre, The geometry of the osculating nilpotent group structures of the Heisenberg calculus
 - Pennig, Ulrich, A noncommutative model for higher twisted K-Theory 1502.02807 to appear in Journal of Topology
 - Baer, Christian; Strohmaier, Alexander, An index theorem for Lorentzian manifolds with compact spacelike Cauchy boundary 1506.00959
 - Deeley, Robin J.; Goffeng, Magnus; Mesland, Bram, The bordism group of unbounded KK-cycles 1503.07398

- Arici, Francesca; Kaad, Jens; Landi, Giovanni, Pimsner algebras and Gysin sequences from principal circle actions 1409.5335 to appear in Journal of Noncommutative Geometry
- Mesland, Bram; Rennie, Adam, Nonunital spectral triples and metric completeness in unbounded KK-theory 1502.04520
- Meyer, Ralf; Pennig, Ulrich, Crossed module actions on continuous trace C*-algebras 1506.01311
- Lesch, Matthias; Moscovici, Henri, Modular curvature and Morita equivalence 1505.00964
- Matassa, Marco, An analogue of Weyl's law for quantized irreducible generalized flag manifolds 1410.8029
- Baum, Paul F.; Dabrowski, Ludwik; Hajac, Piotr M., Noncommutative Borsuk-Ulam-type conjectures 1502.05756 appeared in Banach Center Proceedings
- Carey, Alan; Grosse, Harald; Kaad, Jens, On a spectral flow formula for the homological index 1501.05453
- Takeishi, Takuya, Irreducible representations of Bost-Connes systems 1412.6900
- van den Dungen, Koen; Rennie, Adam, Indefinite Kasparov modules and pseudo-Riemannian manifolds 1503.06916
- Forsyth, Iain; Rennie, Adam, Factorisation of equivariant spectral triples in unbounded KK-theory 1505.02863
- van Erp, Erik; Yuncken, Robert, A groupoid approach to pseudodifferential operators 1511.01041
- Renault, Jean N.; Williams, Dana P., Amenability of groupoids arising from partial semigroup actions and topological higher rank graphs 1501.03027
- Savin, Anton; Sternin, Boris, Elliptic G-operators on manifolds with isolated singularities 1511.01886
- Levitina, Galina; Sukochev, Fedor; Zanin, Dmitriy, Lipschitz properties of smoothed signum for the Dirac operator (in preparation)
- Gayral, Victor; Jondreville, David, Deformation quantization for actions of Q_p^d , 1409.3349 appeared in JFA
- D'Andrea, Francesco; Landi, Giovanni, Quantum weighted projective and lens spaces 1410.4508
- Fathizadeh, Farzad; Gabriel, Olivier, On the Chern-Gauss-Bonnet theorem and conformally twisted spectral triples for C*-dynamical systems 1506.07913
- Barnea, Ilan; Joachim, Michael; Mahanta, Snigdhanayan Model structure on projective systems of C*-algebras and bivariant homology theories

- Mahanta, Snigdhayan, C^* -algebraic drawings of dendroidal sets 1501.05799
- Mahanta, Snigdhayan, Colocalizations of noncommutative spectra and bootstrap categories 1412.8370
- Fathizadeh, Farzad, On the scalar curvature for the noncommutative four torus 1410.8705
- Fathi, Ali; Khalkhali, Masoud, On certain spectral invariants of Dirac operators on noncommutative tori 1504.01174
- Fathi, Ali; Ghorbanpour, Asghar; Khalkhali, Masoud, The curvature of the determinant line bundle on the noncommutative two torus 1410.0475
- Arnlin, Joakim; Wilson, Mitsuru, Riemannian curvature of the noncommutative 3-sphere 1505.07330 to appear in Journal of Noncommutative Geometry
- Neumann, Niels; van Suijlekom, Walter D., Perturbation semigroup of matrix algebras 1410.5961 to appear in Journal of Noncommutative Geometry

Number theory and NCG

- Cornelissen, Gunther; Reynolds, Jonathan, The perfect power problem for elliptic curves over function fields
- Cornelissen, Gunther; Kool, Janne, Edge reconstruction of the Ihara zeta function 1507.03411
- Lorscheid, Oliver, Scheme theoretic tropicalization 1508.07949
- Connes, Alain; Consani, Caterina, Absolute algebra and Segal's Gamma sets 1502.05585