

Report on the Hausdorff Trimester Program

Universality and Homogeneity

September 2 - December 20, 2013

Organizers: Alexander Kechris, Katrin Tent, Anatoly Vershik

Topics

The topics of the program centered on model theoretic and geometric aspects of homogeneity:

- 1) The classification of countable homogeneous structures.
- 2) Connections with model theory, and metric model theory.
- 3) Automorphism groups of homogeneous structures, Ramsey theory and extreme amenability.
- 4) Dynamics of automorphism groups, ergodic theory.
- 5) The universal Urysohn space, universal and related graphs, universal groups.
- 6) Links of invariant measures with the classification of random metrics.

Goals

The perspective of the program was to study and link universality phenomena in very different areas of pure mathematics. These areas included model theory, combinatorics, descriptive set theory, group theory, dynamical systems, especially ergodic theory and topological dynamics, and randomness and stochastic processes. It seems that the program succeeded to promote interactions in an unprecedented way between these different areas.

Universality provides the key ideas for studying many categories and it has become clear that the analogies between the notions of universality in the different areas are very deep. We mention Fraïssé's theory in logic, universal graphs in combinatorics, the universal Urysohn metric space in topology, universality in algebraic geometry and so on.

Organization

We had weekly seminars as well as tutorials, the trimester seminars being of general interest to the program, and individual lectures on more specialized topics. The set-up at the institute was perfect to initiate interactions in many sometimes unforeseen directions. We should mention here that Gregory Cherlin played a very important role in the organization of the Trimester Program. In particular, he was in charge of the seminar program.

During the week of October 28, 2013, we organized a workshop on various aspects of homogeneity. It was attended by about 60 participants, who filled the lecture room to capacity in practically every talk. The excellent lectures by a number of senior mathematicians and many promising young researchers painted a compelling picture of the state of the art in a remarkably broad spectrum of mathematical subjects related to the idea of homogeneity. These included combinatorics, especially graph theory and Ramsey theory, computational complexity, topological dynamics and ergodic theory, probability theory, model theory (including Fraïssé theory, the theory of reducts and metric model theory), permutation groups and the method of forcing in set theory. The workshop showcased exciting recent work that has brought to light intriguing interconnections between these diverse areas of mathematics.

There was also a workshop organized jointly with G. Panina on universality of moduli spaces where interesting interactions became visible from the pure side represented e.g. by M. Kapovich to the more applied side seen in the talks of Sturmfels and I. Streinu.

Results

There are many new interactions which have led to publications in rather unexpected directions, e.g. Tsankov/Evans, proving property (T) for automorphism groups of homogeneous structures via a Tits alternative construction, Nies/Tsankov using metric model theory to prove results on Borel equivalence relations, Ben Yaacov/Glasner/Tsankov using metric model theory to characterize Roelcke precompact groups, Bodirsky/Macpherson/Pinsker proving new results on classifying reducts of ω -categorical structures, Nies/Tent giving short descriptions of finite groups. Nešetřil and his collaborators finished some work on infinite Ramsey theory, Douča started collaborating with Nies on metric structures. At least 30 preprints originated from the program, see the list at <http://www.him.uni-bonn.de/programs/past-programs/past->

~~-trimester-programs/universality-and-homogeneity/publications-pre-prints/~~. Countless others were inspired and initiated by the collaborations started during this period.

There are a number of follow-up conferences continuing the interactions started during the program, in particular in the context of Ramsey theory, ergodic theory, descriptive set theory and metric model theory. For example, we mention the Ramsey Theory Conference, Denver, May 24-28, 2014 and When Topological Dynamics Meets Model Theory, Marseille, June 29-July 3, 2015.