Report on the Hausdorff Trimester Program

# Stochastic Dynamics in Economics and Finance

May 2 - August 23, 2013

**Organizers:** Rabah Amir, Igor V. Evstigneev, Klaus Reiner Schenk-Hoppé

## Topics

Stochastic modeling in economics and finance, in particular stochastic optimization, stochastic dynamic games, and evolutionary modeling.

## Goals

To share new ideas and to develop new approaches to modeling market dynamics and equilibrium in economics and finance with the aim to meet the challenges coming from the economic and financial reality of the 21st century, especially in crisis and post-crisis market environments, by fostering interdisciplinary collaboration between researchers in mathematics, economics and finance.

## Organization

The organization of the trimester programme centered around four complementary themes, each lasting about 1 month and culminating in a workshop. The trimester programme attracted about 90 long-term visitors, including workshop organizers, and up to about 20 additional participants to each of the four workshops.

The first workshop *Stochastic Optimization - Models and Algorithms*, May 27 - 29, 2013, was organized by Alejandro Jofre, R. Tyrrell Rockafellar, and William T. Ziemba. The workshop focused on recent theory and practice in applying stochastic optimization concepts for understanding and solving problems in financial models and economic equilibrium. Each of the three days was organized around a specific topic. The first day began with energy markets in a session oriented to generation/transmission optimization and

pricing mechanisms. This was followed by a sessions on stochastic optimization in terms of risk-deviation-utility, duality and time-consistency, along with other subjects related to financial management. The second day was devoted to portfolio theory and applications, Kelly capital growth theory and applications, and financial market bubbles and crashes. Included in this were issues in portfolio optimization and the pricing of options, as well as capital growth investment criteria. The third day had market equilibrium as its primary focus. The talks were aimed at the philosophy and stability of equilibrium and the ways in which it may be reached or computed. The modeling and estimation of commodity prices got special attention.

The second workshop *Stochastic Dynamic Games with Applications in Economics and Finance*, June 17 - 20, 2013, organized by Rabah Amir, Yuri Kifer, Frank Riedel, and Nicolas Vieille was dedicated to the memory of Jean-Francois Mertens who was a leading figure in game theory, an inspiring scholar to many of the presenters, and a major contributor to this area of game theory. This workshop was devoted to recent advances in the theory of dynamic/stochastic games and its applications in economics and finance. The scope of the workshop was construed very broadly and included in particular discrete-time and continuous-time models. The four days were organized along topical lines. The first day covered recent advances in the basic theory of stochastic games, including repeated games of incomplete information. The second day was dedicated to the general class of stopping games, and included papers dealing with theoretical issues as well as applications to investment and capacity expansion games in economics. The third day dealt with applications of the theory of dynamic games in finance, in particular game options. The fourth day was devoted to stochastic games with uncountable state space including economic applications.

The third workshop *Evolutionary Dynamics and Market Behavior*, July 15 - 18, 2013 was organized by Josef Hofbauer and Sylvain Sorin. It was devoted to recent advances in the theory of evolutionary games and applications. The workshop covered a wide range of approaches, including deterministic and stochastic models, discrete time and continuous time dynamics, games with finitely many players as well as population games. The presentations were devoted to the following themes: Stochastic stability and stochastic learning, in particular limiting properties, on day one. Applications in economics, transportation, networks as well as the connection with learning dynamics on day two. Deterministic procedures (including best response, logit and higher order dynamics) on day three. Specific aspects related to

robustness properties (finite populations, bounded rationality, perturbation, persistence,—) on day four.

The last workshop of the trimester, *Modeling Market Dynamics and Equilibrium: New Challenges, New Horizons*, August 19 - 22, 2013, was organized by Igor V. Evstigneev and Klaus R. Schenk-Hoppé. The focus of the workshop was on new approaches and recent developments in the area of stochastic models for market dynamics and equilibrium. Its main emphasis was on research directions and approaches that could respond to the fundamental challenge of modern mathematics-based economics and finance: the development of new paradigms of theoretical analysis in economic and finance, as well as of market models that would be suitable for practical quantitative applications. The broad scope of the workshop included the following topics: behavioral and evolutionary models of markets, game theory and information; ambiguity and investment, risk management and trading; decentralized trade and random matching; Von Neumann-Gale dynamical systems; and detection problems and their applications in finance.

In addition to the many ad hoc activities, five lecture series were given:

*Optimal stopping problems: Basic formulations, concepts and methods of solution* (Albert Shiryaev and Mikhail Zhitlukhin),

*The Monge-Kantorovich Problem* (Alexander Kolesnikov),

*Unbeatable Strategies* (Yurii Khomskii),

*Introduction to the theory of evolutionary games* (Josef Hofbauer and Sylvain Sorin), and

*Introduction to the theory of supermodular optimization and games* (Rabah Amir).

## Results

We now provide a brief summary of selected results and completed papers that were obtained or significantly advanced during this trimester program. Our aim is to highlight some of the work in stochastic dynamical modeling in economics and finance that came to fruition during the program. We stress that the three areas chosen cover only part of the many innovative research ideas that emerged from the gathering of a diverse group of mathematicians, economists and scientists from other disciplines at HIM.

**Stochastic/Dynamic Games**

In the area of stochastic/dynamic games, the workshop witnessed quite a diverse set of research lines with quite some success on several fronts. He and Sun (2014) settled a long standing open problem by provising sufficient conditions on the existence of Markov perfect equilibrium in discounted stochastic games with uncountable state and actions spaces. Hörner, Klein and Rady (2013) made significant progress on their work on continuous time bandit problems. Sorin and Vigeral (2013) provide a new perspective on the existence of the limit value in two person zero-sum discounted repeated games. Renault, Solan and Vieille (2014) develop a novel analysis of a dynamic version of the well known strategic information transmission model. Renault and Ziliotto (2014) study limit equilibrium payoffs in hidden stochastic games. Finally, Benssousan and Yam (2013, a-d) completed a series of four papers on the emerging topic of mean field games, some theoretical/foundational and some dealing with a novel application of this class of games to finance.

**Game Theory (with Applications)**

In the general area of game theory and applications to market behavior, which is an integral part of each of the last three areas listed above, several projects were completed. Soza and Tourki (2013) and Grant, Soza and Tourki (2013) made substantial progress on their fundamental work on the existence of equilibrium in Bayesian games with non-monotone solutions, including the proof of a significant new version of Cellina's fixed point theorem. De Castro (2013) provides an important generalization of the concept of affiliation used in auction theory and Bayesian games. Amir and De Castro (2013) propose a new class of strategic games, those with strategic pseudo-complementarities, that always possess pure strategy Nash equilibria, as an application of Tarski's intersection point theorem. He and Sun (2014) extended the scope of classical purification results for games with incomplete information. Finally, Khan, Rath, Sun and Yu (2015) provide an insightful study of strategic uncertainty and randomization in large games.

For evolutionary game theory and learning in games, Hofbauer, Balkenborg and Kuzmics (2015) introduce the refined best-response correspondence in normal form games and investigate its properties. Hofbauer and Zhang (2014) were able to obtain new results on equilibrium selection via replicator dynamics in the simple class of 2x2 coordination games.

As for applications to market behavior, Amir, De Castro and Koutsougeras (2014) generalize the classical results by Mankiw and Whinston

on the comparison between free entry and socially optimal entry. Amir and Burr (2015) investigated the effects of corruption in entry certification on market entry by firms. Amir, Encaoua and Lefouili (2014) show that, for patents of uncertain validity, the standard comparative study of fixed fee versus royalties yields a reversal of the usual conclusions under very general assumptions.

**Risk-Sensitive Investment Management**

Mathematical theories of optimal asset allocation based on the so-called risk-sensitive optimal control theory were the focus of Mark Davis and Sébastien Lleo's work during their stay at the Hausdorff Research Institute for Mathematics. The risk-sensitive approach has been studied in the control theory literature for many years, see for example the textbook by Whittle (1990). It is a generalisation of classical stochastic control in which the degree of risk aversion or risk tolerance of the optimising agent is parameterised explicitly in the objective criterion. Applications to optimal investment were pioneered by Bielecki and Pliska (1999), and the approach has major advantages in providing combining the optimality criteria of Markowitz (1952), Kelly (1956) and Merton (1969, 1971) in a unified package in which the investor's attitude to risk is summarised in one parameter $\theta$ which quantifies the investor's reward vs. risk trade-off.

The work by Davis and Lleo extends the risk-sensitive approach to Asset and Liability Management (ALM). Effective ALM models are crucial for funded investors such as endowment funds, pension funds and insurance companies, but also for investors such as banks and hedge funds who have the ability to grow their asset base by borrowing. They proposed a solution for an ALM problem in a jump-diffusion setting under two sets of assumptions. Under both sets of assumptions, the asset prices and liability value depend on a random factor process $X_t$, the components of which can be interpreted either as macroeconomic factors or simply as a statistical representation of the uncertainty of asset returns. Under the first set of assumptions, the growth rate of assets and liabilities are affine functions of the factors, while the diffusions are constant. The factors are modelled as Gaussian diffusion processes. Under the second set of assumptions, both the growth rate and diffusion depend on the factors, making it possible to incorporate stochastic volatility. This article has now appeared in the journal *OR Spectrum* in July 2014, (David and Lleo, 2014b).

Davis and Lleo also wrote most of the first draft of a book on Risk-

Sensitive Investment Management (RSIM) during their stay at the HIM. The book considers theoretical developments in stochastic control theory in a diffusion and jump-diffusion setting and investment applications to benchmarked portfolios, as well as the new results on ALM. This book was published by *World Scientific Publishing* in September 2014 (Davis and Lleo 2014a).

**Change-point Detection and Optimal Investment**

Shiryaev, Zhitlukhin and Ziemba (2015) developed a framework of analysing bubbles in stock markets and moments they burst based solely on the stock prices. Their research concerns two issues: first, is it possible to predict stock market crashes and, secondly, could investors have exited the market unscathed with most of their gains or actually shorted the market successfully? Theoretical results are verified by empirical observations based on particular market crashes. The research forms part of Zhitlukhin (2014)'s PhD thesis.

The first part of the research studies the Bond Stock Earnings Yield Differential (BSEYD) model proposed by Ziemba, which compares current bond and stock prices/earnings ratios and treats the difference as a signal of an approaching crash (when it exceeds an upper threshold). This model produces exceptional results on empirical data by correctly predicting 12 out of 12 crashes in the data analysed.

The second part develops around models to detect abrupt changes in stochastic processes. Such changes can correspond to the moment when a bubble bursts. The fast detection of such changes is essential in protecting a portfolio from incurring substantial losses. The authors formulate a model of a stochastic process with drift changing at an unknown moment and solved the problem to find the stopping time closest to that moment with respect to a particular criterion. Applied to the empirical data, the model performed well, correctly exiting the market losing no more than 10-20% of the top price.

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