

Report on the Hausdorff Trimester Program

Integrability in Geometry and Mathematical Physics

January - April 2012

Organizers: Franz Pedit, Ulrich Pinkall, Iskander A. Taimanov, Alexander P. Veselov, Katrin Wendland

Topics

Classical and quantum integrable systems, applications in geometry, mathematical physics and representation theory.

Goals

To share results and to discuss recent progress and new directions of research in the theory of integrable systems in mathematics and theoretical physics.

Organization

The program started with the workshop *Integrability-modern variations* organized by A. Veselov and K. Wendland on January 9-13, 2012. The main part were the mini-courses on certain modern aspects of the theory of integrable systems given by the leading experts in the area (B. Dubrovin, G. Felder, I. Gordon, R. Thomas, A. Varchenko). These courses were followed by three afternoon discussion tables, which proved to be a very useful format for the exchange of ideas coming from a number of different areas such as algebra, geometry and mathematical physics. The speakers put a lot of effort into explain the cutting-edge research in ways accessible to all the participants, including PhD students.

A special conference on *Geometric PDE and integrability* was organized on January 23-25 by M. Haskins and I. Taimanov. the main goal of this meeting was to discuss possible applications of integrable systems in partial differential equations of geometric origin. Some of the equations presented

are known to be integrable and this fact helps substantially in finding explicit solutions. Other equations have been shown to be related to integrable systems but need a more elaborate study in order to obtain a better understanding of their solutions. The talks covered a number of topics, such as conservation laws, conformally invariant functionals, hyper-Kähler and Ricci-flat manifolds.

During March 26 - 30, 2012, F. Pedit and U. Pinkall together with M. Rumpf from the University of Bonn organized a workshop with the title *Navigating the Space of Surfaces*. The goal was to bring together three different scientific communities that all are heavily involved with the geometry of surfaces: one group of participants studied surfaces from the viewpoint of integrable systems, another group saw surfaces from the perspective of geometry processing for computer graphics and finally there were participants that look at surfaces as presenting interesting challenges for (theoretic or numerical) analysis. The workshop was a great success: from each group we had a substantial subset of the internationally leading experts of the respective fields. The variety of viewpoints was taxing but highly inspiring for the participants.

A common thread of the workshop concerned Willmore surfaces (coincidentally it was during the month of the workshop that a proof of the Willmore Conjecture was published by F. Marques and A. Neves). One of the immediate outcomes during the workshop was the first algorithm that allows to numerically search for minimizers/stable critical points of the Willmore functional in each conformal class. This work was later presented as a paper by three of the participants in the most renowned conference in computer graphics (SIGGRAPH 2013 Conference Proceedings).

The final workshop of the symposium was titled *Integrability in topological field theory*, and was organized by A. Klemm and K. Wendland on April 16-20, thus including a local expert from Bonn as an organizer. The program was equally balanced between speakers from mathematics and from physics. This workshop was devoted to classical as well as recent developments around the mathematics of quantum field theory and integrable systems, in most cases related to the properties of partition functions of quantum integrable systems. In particular, recent developments concerning the so-called topological recursion relations were central. Key topics included knot invariants and Gromov-Witten invariants, geometric quantization of cluster varieties, modularity, the geometric relation of vertex algebras to the period integrals of hypersurface singularities, and the role of quantum cohomology in the

description of quantum integrable systems, to name just a few. The workshop was followed by an Oberwolfach workshop on “Singularity theory and integrable systems”, April 22-28, organized by B. Dubrovin, C. Hertling, I. Strachan and K. Wendland, which was attended by five of the participants of the symposium.

Besides those four workshops regular weekly talks, more informal seminars on a daily basis and ad hoc discussion sessions took place. These activities, usually scheduled with the traditional afternoon cake and coffee as a break, added significantly to the integration of and communication between the various fields and visitors to HIM. It also gave Ph D students and Post-Docs an avenue to present and discuss their work in front of an international audience of experts.

Results

The homepage of the program lists 19 publications that were completed during the symposium, but the actual number is known to be bigger. Here are some results found during the program.

1. The spectral data and loop algebra valued meromorphic connection for the Lawson minimal surface of genus two was found.
2. A relation between the Calogero-Moser system and elliptic solitons from one side and minimal surfaces in the three-space, i.e. minimal tori with planar ends, was established. It appears that the spectral curves for such tori are at the same time the spectral curves of certain elliptic solitons.
3. A geometric formulation of the Novikov-Veselov flows in terms of conformal flows of surfaces in space was developed and applied for the first time to surfaces of genus zero.
4. A proof in the spirit of K. Uhlenbeck’s seminal paper on “Harmonic maps into Lie groups: classical solutions of the Chiral model” for the quantization of the harmonic map energy for maps of 2-spheres into pseudo-Riemannian symmetric spaces with applications to Willmore spheres were found.
5. An approach to Chern-Simons theory based on Vogel’s notion of universal Lie algebra was developed.

6. A relation between the separation coordinates on the sphere and the real version of the Deligne-Mumford-Knudsen moduli was established
7. An explicit construction of commuting ordinary differential operators of higher rank was proposed.
8. The stability theory for the bi-Hamiltonian systems is developed.

The symposium was also attended by a number of graduate students, a fact which added to the lively research atmosphere. For example, A. Varchenko attended the entire symposium as a senior scientist, and with his student D. Wright, who also visited the Hausdorff Institute during the symposium, he completed a project on critical points of master functions and integrable hierarchies during the symposium.

A number of new collaborations arose from the scientific discussions at the symposium. For example, Ch. Korff attended the symposium during two separate weeks and has begun a new project on the interpretation of topological quantum field theories arising from WZW models in terms of singularities with K. Wendland. A. Taormina visited the MPI for Mathematics during the symposium and together with her coauthor K. Wendland began a new collaboration with D. Zagier, where they formulated a conjecture on the modular properties of counting functions in conformal field theory, which they were able to prove in a number of cases. A. Veselov was involved in active collaboration with a number of participants (L. Aguirre, M. Feigin, M. Hallnas, R. Inoue, K. Schöbel), which led to two papers completed and one in preparation. F. Pedit and U. Pinkall started new joint projects with F. Burstall and C. Bohle (fostered by discussions with A. Veselov and P. Grinevitch) on a geometric interpretation of the Novikov-Veselov hierarchy. F. Pedit, F. Burstall and J. Dorfmeister worked on the quantization of harmonic map energies in pseudo-Riemannian symmetric spaces. I. Taimanov collaborated on a beautiful relation between elliptic solitons and minimal surfaces with C. Bohle.