Algebraic models of Galois deformation spaces

Deformation spaces for Galois representations of a p-adic field with p-adic Hodge theoretic conditions imposed play a fundamental role in recent progresses on the Langlands program and its mod p and p-adic variants. The global geometry of such spaces appear to be complicated in general, and the Breuil-Mézard conjecture quantifies this complexity in terms of (modular) representation theory.

In this series of talk, we explain the construction, following the on-going work [LLLM2], of certain nice finite type schemes whose p-adic completion provide charts for (portions of) the Emerton-Gee stack, and in particular whose completion at closed points recover some potentially crystalline deformation spaces. These models are certain subschemes of the global Schubert varieties of Pappas-Zhu(which occur in the theory of local models of Shimura varieties), and are degenerations of flag varieties into deformed affine Springer fibers. Their geometry and combinatorics thus provide a basic link between the Emerton-Gee stack and (modular) representation theory. We will also discuss the application of these models to the geometric Breuil-Mézard conjecture and the weight part of Serre's conjecture, which were the motivations for their construction.

A tentative plan of the talks are as follows:

1. Talk 1: Motivations: The weight part of Serre's conjecture, the Breuil-Mézard conjecture. Overview of the course.

   Related reading: [GHS] (especially the introduction), [H].

2. Talk 2: Structure of Kisin modules with generic descent data, relation with Pappas-Zhu local models and stacks of étale φ-modules . Examples for GL_2 and GL_3.

   Related reading: [PZ], [CL] (for Pappas-Zhu local models), [LLLM1, §4] , [LLL, §3].

3. Talk 3, 4: Construction of the models and their geometric properties. Relationship with potentially crystalline deformation rings and the Emerton-Gee stack.

4. Talk 5: Global applications.

References

[H] F. Herzig, *The weight in a Serre-type conjecture for tame n-dimensional Galois representations*, Duke Math J. **149**, no. 1 (2009).

[PZ] G. Pappas and X. Zhu, *Local models of Shimura varieties and a conjecture of Kottwitz*, Invent. Math. **194** (2013), no. 1, 147–254.

[CL] A. Caraiani and B. Levin, *Kisin modules with descent data and parahoric local models*, Ann. Sci. Éc. Norm. Supér. (4) **51** (2018), no. 1, 181–213.

[GHS] T. Gee, F. Herzig and D. Savitt, *General Serre weight conjectures*, Journal of the European Math Society 20.12 (2018).

[LLLM1] D. Le, B. Le Hung, B. Levin, and S. Morra, *Potentially crystalline deformation rings and Serre weight conjectures: shapes and shadows*, Invent. Math. **212** (2018), no. 1, 1–107.

[LLL] D. Le, B. Le Hung, and B. Levin, *Weight elimination in Serre-type conjectures*, Duke Math. J., to appear.

[LLLM2] D. Le, B. Le Hung, B. Levin and S.Morra, *Local models for Galois deformation rings and applications*, in prepa

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